

Summary

In order to protect the bees, we must first decide which factors have the greatest impact on honeybee colony size. We used a ordinary differential equations to model the populations of drones and workers. We expressed the eclosion rate of honey bees as a function of the number of worker bees, the egg laying rate, the protein storage function and the pesticide mortality rate. The fertility rate determined the gender of the brood. The mortality rate was the function of the seasonal background mortality rate, the pesticide mortality rate and the protein storage function.

After pesticides were applied to an area, its effective dosage with respect to time could be modeled with a radioactive decay function which we were able to work out using its half-life. We then assumed that each bee had a threshold mortality rate that obeyed the logistic distribution, so we used a sigmoid function to express the cumulative distribution of bee mortality at that time. We used matrixes to convert the number of flowers into the amount of essential amino acids and obtained the protein storage function for each of the ten types of amino acids. When the protein storage function of a type of amino acid dropped to zero, we calculated the number of bees that could not get enough nutrition and put them together to get the total mortality rate caused by food shortage.

We did a sensitivity analysis on our model by varying the crop variety, egg laying rate, fertilization rate, pesticide usage and background mortality rate in a certain colony. We found that while a lack of crop variety, a low egg laying rate and a low fertilization do cause the colony's population to decline, the colony's population was steady over the four years. When the background mortality rate of bees increased to above $\frac{1}{30}$, or when strong insecticides such as fipronil was used, colony collapse disorder was observed.

Furthermore, we modeled and predicted the number of hives required to sustain the pollination of a 20-acre field. We considered the factors affecting the number of foragers emerging from the hives such as wind speed, precipitation, location, solar radiation, and outdoor temperature. We agreed that temperature is the most important factor among all. We later employed data from earlier researches and utilized a cubic polynomial function to analyze the functional relationships between the number of bees emerging from the hive every thirty seconds and outdoor temperature. After taking the mean of this continuous function, we received the relative rate of difference between the calculated mean and emerging bees. This calculation allowed us to transform the speed-related measurement bees-per-minute into quantitative numerals related to the number of bees.

Then, we exemplified several crops and acquired their average number of flowers per acre. Ultimately, knowing the number of flowers a honey bee visits in a day and having the predictive data of working bee population, we are able to show that approximately 20 hives are required to pollenate 20 acre of crops. However, when the crop density is high and the climate condition in that region is harsh, approximately 60 hives are needed.

Keywords: ordinary differential equations, flower-protein matrixes, radioactive decay function, sigmoid function, non-linear regression

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We're Losing Honeybees!

AND NOT JUST FOR HONEY.....

Did you Know?

About **80%** of all flowering plants depend on honeybees for pollination, but the population of honeybees has recently shown a significant decline.

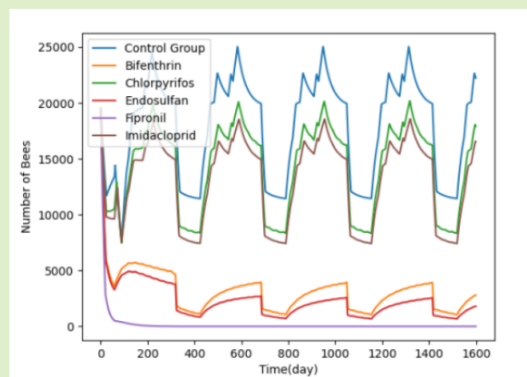
Main Causes of Bee Decline

- Our research has shown that bee decline can be attributed to factors such as protein inadequacy, pesticide usage, and viruses.
- The primary factor attributed to bee decline is pesticides.



What should we do?

Reduce the amount of pesticides (especially Imidacloprid) sprayed onto crops during pollination seasons.



This graph shows the relationship between bee population and pesticide usage.



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1 Introduction

1.1 Background

Honey bees are both ecologically and economically indispensable animals that serve vital roles in crops pollination, maintenance of balance in the ecosystem, as well as honey production. 70 percent of cultivated species in tropical areas rely on honey bees for pollination, and it is widely-accepted that the production of various fruits, seeds, and nuts will decrease by 90 percent if honey bees become extinct. Furthermore, honey bees are vital to the balance of the ecosystem. By helping to regenerate trees, they are important for conserving biodiversity in the ecosystem, as well as protecting the food chain.

However, despite the prominence of the role of honey bees, threat to honey bee population has been raised in the past 20 years. The term CCD(Colony Collapse Disorder) was created to name the abnormal global honey bee decline. Past research has shown that bee decline can be attributed to a variety of factors including viruses, predators, pesticides, habitat destruction, and more.

1.2 Problem Restatement

Honey bees are important pollinators in modern agriculture. Without bees, many plants would die and the whole ecosystem would be endangered.

Recently, the population of bees are declining in different locations all around the world and the term Colony Collapse Disorder(CCD) was created to describe such a phenomenon. In order to protect the bees and save our ecosystem, we must first identify the what caused CCD.

In order to understand how different work factors contribute to CCD, we will build a model that predicts the population of a honeybee colony over time by calculating the birth rate and death rate of bees under different circumstances. We will investigate how natural reasons, such as the fertility rate of a colony, and human reasons, such as pesticides, contribute to the population dynamics of honey bees. Then, by doing a sensitivity analysis on our model, we will identify what caused CCD and what makes a colony healthy.

The purpose of this model is to help evaluate different solutions to protect the honey bees. There will also be a non-technical blog that offers insights about how people could protect the bees.

2 Assumptions and Justification

- **Assumption 1:** The difference between a hive bee (female) and a forager (female) is negligible.
- **Justification 1:** Foragers and hive bees serve different roles in a colony. However, a bee can easily make a transition from a forager to a hive bee or vice versa. As soon as foragers die, hive bees quickly get recruited to compensate for the lack of foragers. When there are too much foragers, some foragers get sent back to be hive bees. Thus, whenever the population of one group increases, the population of the other group increases accordingly and the difference between the two could be neglected.
- **Assumption 2:** Pesticides have very limited effects on drones.

- **Justification 2:** Since drones mostly stay in the hive, they do not contact the flowers directly. Pesticide particles in the air do affect them, but this effect is approximately ten times smaller than the pesticides' effect on foragers. Thus, our model assumes that pesticides do not effect drone bees.
- **Assumption 3:** The colony does not share its hive with another colony.
- **Justification 3:** Naturally, each hive houses only one colony. Sometimes, beekeepers put two colonies in the same hive. This condition is not within the scope of this model.
- **Assumption 4:** A parcel of land is a rectangular field.
- **Justification 4:** Crop lands are normally rectangular. Small variance in shape does not impact our model greatly.
- **Assumption 5:** A hive consists a single colony.
- **Justification 5:** According to assumption 3, assumption 5 stands.

3 Variables

See the next page.

4 Population Model

4.1 Basic Model

Our model predicts the population of female (worker) bees W and male (drone) bees D . By adding them together, we obtain the total number of bees in a colony N at day t . We mainly focus on the population dynamics of the worker bees, as the drone bees do not contribute to colony production outside mating season. According to the analysis above, we obtain the following equation:

$$N(t) = W(t) + D(t)$$

The rate of change in the number of worker bees equals its eclosion rate E_w minus its death rate M , in which M is the sum of mortality caused by protein shortage mF and mortality caused by the usage of pesticides mP . That is,

$$\frac{dW}{dt} = E_w(t) - M$$

and

$$M = mF(t) + mP(t)$$

Background mortality in different seasons are also different. In the summer when workload is

Table 3.1: Variables Table

Variables	Description
W	the population of female (worker) bees
D	the population of male (drone) bees
N	the total number of bees in a colony
t	time in days
$E(t)$	eclosion rate
M	total mortality rate
mF	mortality rate caused by protein shortage
mf_i	the mortality rate caused a shortage of amino acid i
mP	mortality caused by the usage of pesticides
mS_w	seasonal factor of mortality of worker bees
mS_d	fall mortality of drone bees
$L(t)$	egg laying rate
nf_i	the storage functions of different essential amino acids
f	fertilization rate
P_e	amount of essential protein produced by a type of crop
N	the number of a certain type of flower within the forage range of a colony
P	the correlation between flower species and the amount of different amino acids they produce
N	the number of the four types of flowers that blossomed on a given day
A	the level of essential amino acid storage on a given day
C	the amount of each amino acid a honey bee or honey bee larvae consumes
N_0	initial dosage of pesticide
u	number of bees emerging from a hive per minute
T	temperature measured in degrees Celsius
$f(T)$	equals u
M_v	mean value of u
T_0	a given temperature
S_a	Alfalfa flowers per acre
S_d	Date Palm flowers per acre
S_{s1}	Summer Squash flowers per acre
S_{s2}	Sunflower seeds per acre
T_a	temperature where Alfalfa flowers pollinate
T_d	temperature where Date Palm flowers pollinate
T_{s1}	temperature where Summer Squash flowers pollinate
T_{s2}	temperature where Sunflowers pollinate
q_a	the number of hives needed to pollinate a 20-acre land planted with Alfalfa
q_d	the number of hives needed to pollinate a 20-acre land planted with Date Palm
q_{s1}	the number of hives needed to pollinate a 20-acre land planted with Summer Squash
q_{s2}	the number of hives needed to pollinate a 20-acre land planted with Sunflower

heavier, bees have a shorter lifespan. We can represent this difference with a seasonal factor mS_w , and add it to our sum of mortality:

$$\frac{dW}{dt} = E_w(t) - mF(t) - mP(t) - mS_w(t)$$

Similarly, we could express the rate of change in drone bees:

$$\frac{dD}{dt} = E_d(t) - mF(t) - mS_d(t)$$

4.2 Eclosion

The total birth in a colony in a given day $E(t)$ is a function of the egg laying rate $L(t)$, the storage functions nf_i of different kinds of essential amino acids and the number of worker bees in the colony W . The larger the number of food storage and the number of worker bees, the higher the eclosion rate is. To simplify the model, we express this relationship with a simple function that increases from 0 to 1 as W and nf_i gets larger. Pesticides also increase the mortality rate of the colony. We write the number of total births as:

$$E(t) = L(t) \left(\frac{W}{W + \beta} \right) (1 - mP(t)) \prod_{i=1}^4 \frac{nf_i}{nf_i + \alpha}$$

For this model, we chose $\alpha = 3.2 * 10^6$ and $\beta = 10000$. If an egg is fertilized, it later becomes a female (worker) bee. If it isn't, it grows into a male (drone) bee. Let f be the fertilization rate and we obtain the following equations:

$$E_w(t) = fE(t)$$

$$E_d(t) = (1 - f)E(t)$$

4.3 Protein Variance and Abundance

Honey bees feed on a wide variety of crops. We chose the following crops to represent other crops because the availability of data about the nutrition values of their pollen. These plants also represent plants with different flowering seasons. In the flowering season of each crop, the amount

Table 4.1:

Crop	Flowering Season	P_e/P
Alfalfa	April	20.23
Date Palm	March	19.77
Summer Squash	July	16.39
Sunflower	June	15.19

of essential protein P_e produced by that type of crop can be expressed as:

$$P_{ei} = NC_p \frac{P_e}{P}$$

N is the number of flowers within the forage range. C_p is the amount of protein each flower produces and $\frac{P_c}{P}$ is the amount of protein(g) per 100g of pollen. According to Cralsheim [1, 3], an adult bee requires 3.4-4.3g of pollen everyday and a larva requires 120-185g of pollen every day.

Bees require ten types of protein to survive. The table above shows how much of the ten essential types of protein a certain type of flower contains. The following matrix $\mathbf{P}$ expresses the relationship between the flower species and the types of essential amino acids. $m = 1,2,3,4$ corresponds to alfalfa, date palm, summer squash and sunflower respectively and $n = 1,2,3,4,5,6,7,8,9,10$ corresponds to arginine, histidine, isoleucine, leucine, methionine, phenylalanine, threonine, tryptophan and valine, respectively. The flower m produces $A_{m,n}$ g of essential amino acid n per 100g of pollen. This data is obtained from Somerville [9] and Taha [11].

$$\mathbf{P} = \begin{pmatrix} 1.55 & 2.12 & 1.75 & 2.56 \\ 1.71 & 1.62 & 3.83 & 4.00 \\ 3.56 & 3.30 & 3.54 & 3.04 \\ 6.38 & 6.14 & 7.14 & 6.35 \\ 3.15 & 4.51 & 4.68 & 4.98 \\ 0.22 & 0.18 & 0.43 & 0.24 \\ 1.51 & 1.53 & 1.57 & 0.76 \\ 2.34 & 2.45 & 2.44 & 3.41 \\ 0.60 & 0.61 & 0.63 & 0.33 \\ 4.92 & 4.61 & 5.23 & 5.46 \end{pmatrix}$$

Let the number of the four types of flowers that blossomed on a given day be

$$\mathbf{N} = (aa_1 \quad aa_2 \quad aa_3 \quad aa_4)$$

and let the level of essential amino acid storage on a given day be

$$\mathbf{A} = (nf_1 \quad nf_2 \quad nf_3 \quad nf_4)$$

The increase in the amount of each essential amino acid available on a given day could be calculated using the formula $\mathbf{NP}^T$.

According to Groot [8], on each day, the amount of each amino acid a honey bee or honey bee larvae consumes can be expressed is

$$\mathbf{C} = (3.00 \quad 1.50 \quad 4.00 \quad 4.50 \quad 3.00 \quad 1.50 \quad 1.50 \quad 1.50 \quad 1.00 \quad 4.00)$$

Thus, the rate of change of the ten amino acids is

$$\partial \mathbf{A} = \mathbf{NP}^T - (W + D)\mathbf{C}^T$$

If a colony lacks any of the ten types of amino acids, its mortality rate would increase dramatically. Thus, the mortality rate $mF(t)$ satisfy:

$$mF(t) = 1 - \prod_{i=1}^4 1 - f_i(t)$$

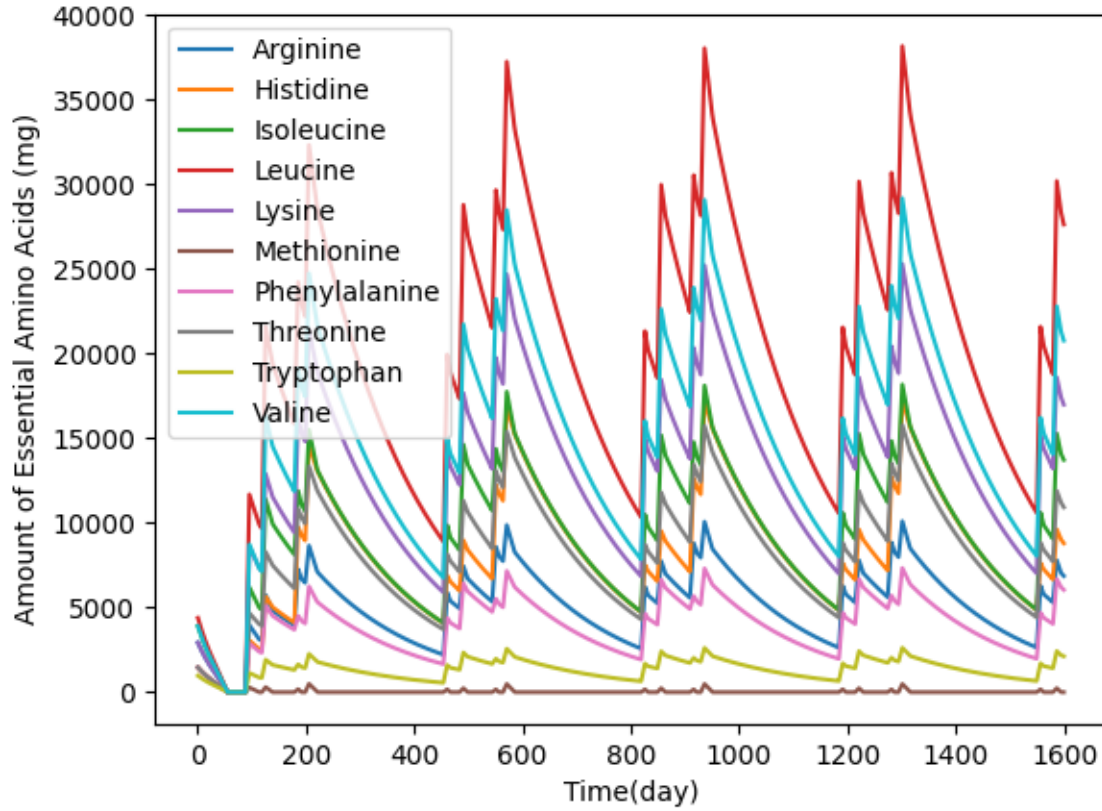


Figure 4.1: the storage functions of different essential amino acids, nf_i

in which $f_i(t)$ is the mortality rate caused a lack of amino acid i . As long as the food storage function $f_i(t)$ is positive, no additional deaths is caused. When there is a forage shortage, suppose the colony ensures that a maximum number of bees get enough nutrition. So the amount of bees that don't get enough nutrition equals the nutrition deficit $-nf_i$ divided by the required amount of that type of amino acid. Those bees die. Thus,

$$f_i(t) = \begin{cases} 0, & \text{if } nf_i > 0 \\ -\frac{nf_i}{C_i(W+D)}, & \text{if } nf_i \leq 0 \end{cases} \quad (4.1)$$

4.4 Pesticides

A pesticide's effective dosage declines after it is applied to crops. We assume that honeybee mortality decreases evenly over time. After applying an initial dosage N_0 to crops at time t_0 , the pesticide's effective dosage at time t_0 can be expressed in the function:

$$N(t) = N_0 e^{\lambda_i(t-t_0)}$$

Since the half-life hl_i of different pesticides is given in Migdal's paper [14], we can calculate λ_i using $\lambda = \frac{\ln 2}{hl_i}$

Pesticide	Half-life	LD_{50}
Bifenthrin	87	0.015
Chlorpyrifos	50	0.072
Endosulfan	86	0.014
Fipronil	142	0.007
Imidacloprid	174	0.061

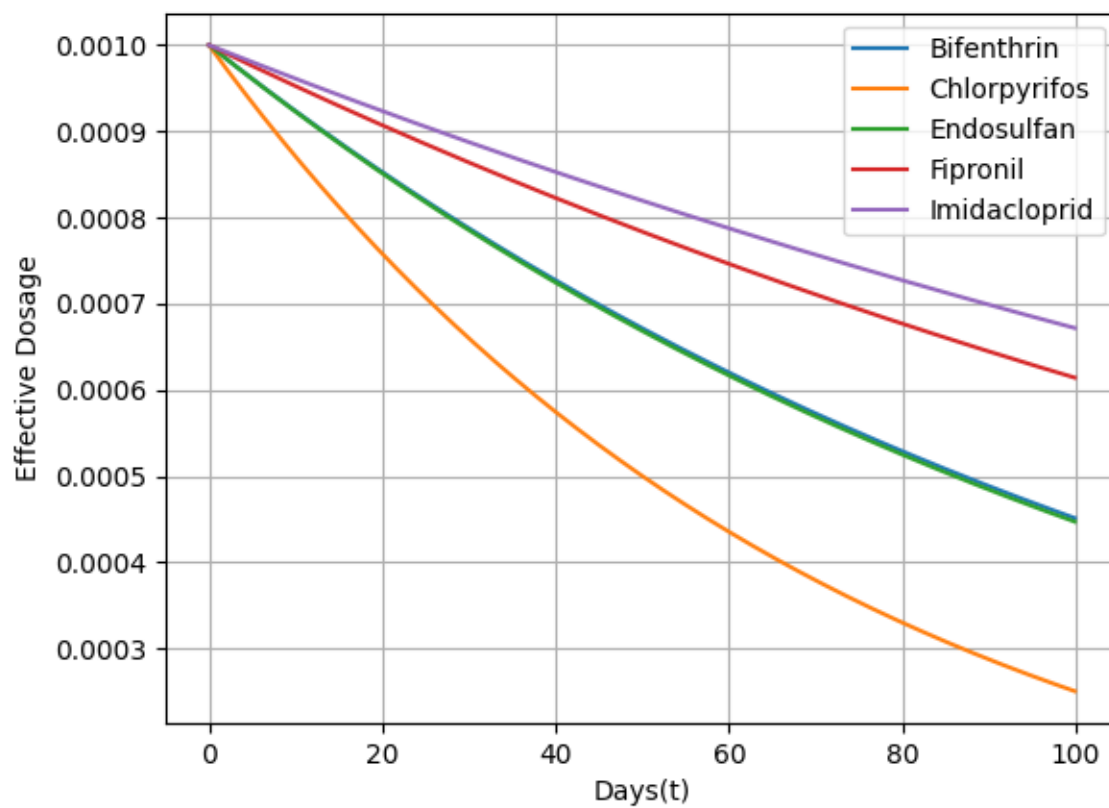


Figure 4.2: The effective dosage per 0.001g of pesticide

We assume that each bee has a different lethal dosage and that the distribution of the lethal dosage of bees in a certain colony obey the logistic distribution. Integrating such distribution, we could use the sigmoid function to represent the colony's mortality rate. The relationship between the mortality $mP(t)$ and the pesticide dosage could be expressed as:

$$mP(t) = \frac{2}{1 + e^{-kN(t)}} - 1$$

LD_{50} , the dosage required to kill half of the bees, is shown in table 2, so we can calculate k using $k = \frac{\ln 3}{LD_{50}}$

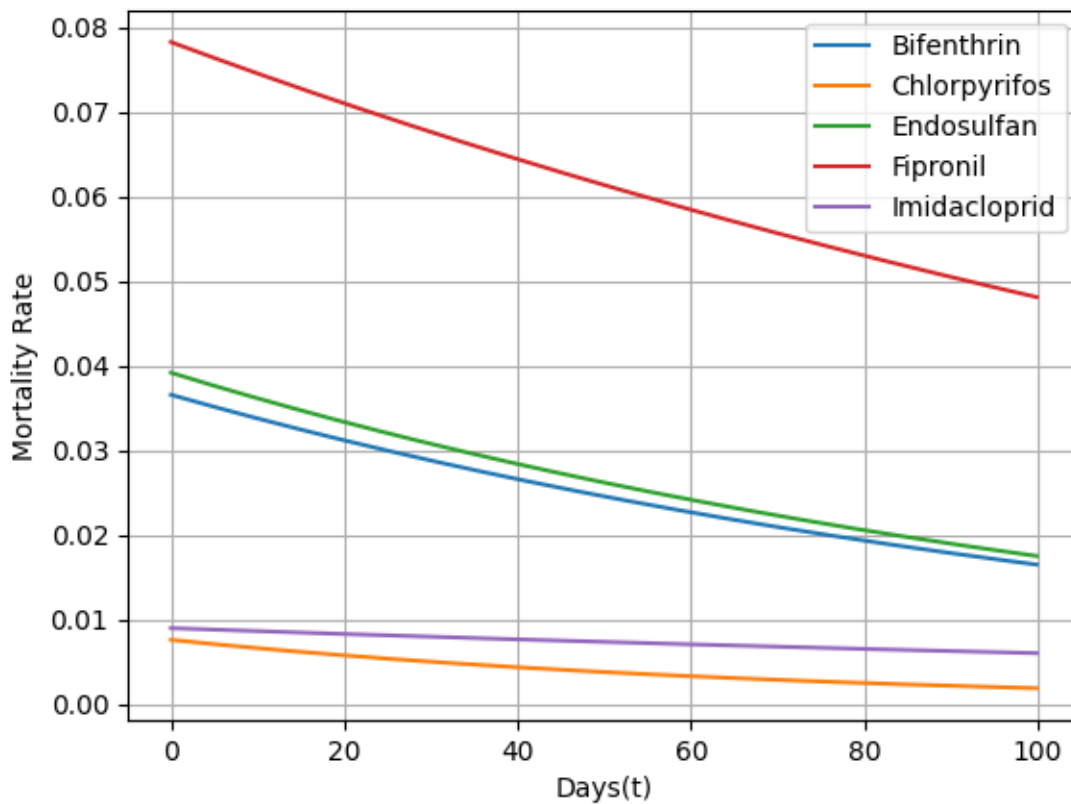


Figure 4.3: The mortality rate caused by pesticides

4.5 Seasonal Factor

During spring and summer, foragers are very active as they need are constantly flying from flower to flower searching for newly blossomed flowers. Due to their high level of activity, they have a shorter lifespan in those months. During autumn and winter, on the other hand, forages are not as active. As a result, they may live longer.

Many factors may contribute to an increase in the honeybee's level of activity, such as the the distance between flowers, the wind speed of the area and the location of the hive. To simplify our model, we simply assume that the seasonal background mortality is constant.

Suppose a colony has a constant background mortality rate mS_w and every day, the oldest members of the colony die. For each bee, it will die on the $\frac{1}{mS_w}$ th day. Thus, the average lifespan in that colony will be $\frac{1}{mS_w}$ th.

Since the average lifespan of honey bees in the summer is 6 weeks in spring and summer, and 5 months in autumn and winter, the background mortality rate mS_w is $\frac{1}{36}$ in spring and summer and $\frac{1}{152}$ in autumn and winter.

In fall, most drone bees die after having sexual intercourse with the queen causing the fall mortality of the drone bees to increase dramatically [12]. The remaining drones get expelled from the cave. When modeling the population of a bee colony, we consider the latter event equivalent with the former. We let the increased mortality rate mS_d be $\frac{1}{50}$ in this model so that the drone bees' population decline rapidly in autumn.

5 The Model Results

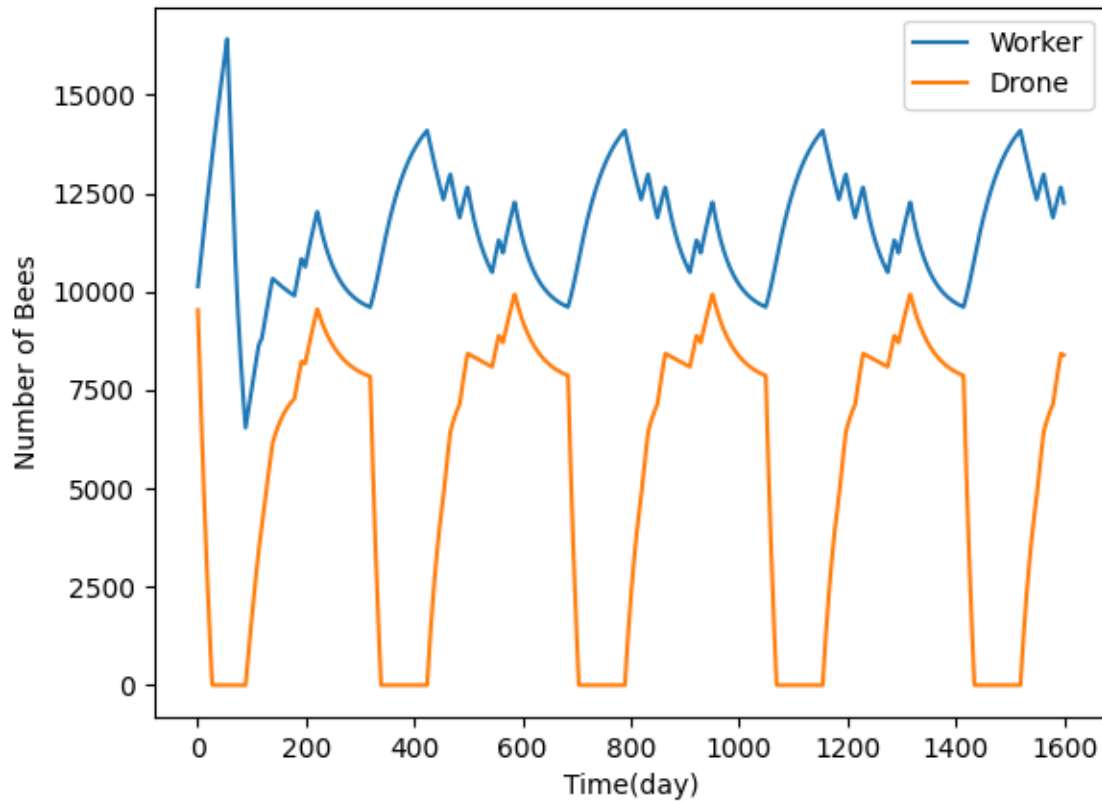


Figure 5.1: Population model of healthy colony

According to our model, this is the population of a healthy bee colony with a fertilization rate > 0.5 . As seen from the figure, due to the abundance of protein, the population of worker bees increase in the summer despite a larger seasonal mortality rate. Their population declines when

they nearly run out one types of flower but rebounds when another type of flower blossoms. The population of drones drop to zero every winter because drones all died or get expelled from the colony. The population curve of both the workers and the drones are similar from year to year.

6 Sensitivity Analysis

6.1 Crop Variety

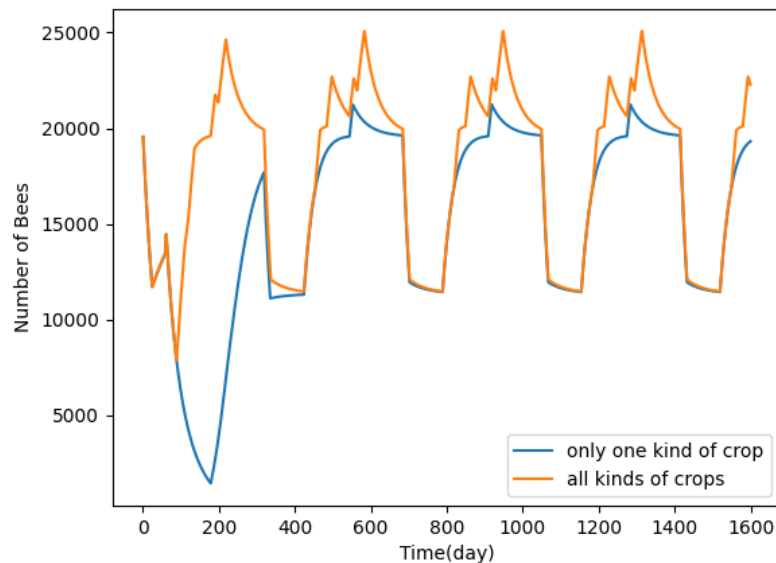


Figure 6.1:

We change the number of crop kinds: one group with four different crops with 100,000 each, the other with 400,000 of the same crop. If the bees only have access to one single kind of crop, our graph shows that the number of bees is about the same in winter but has a lot of difference (about 5,000 bees) in summer. Their unbalanced diet leads to less essential acids, and thus causes more deaths. Also, we observe that bees relying on one kind of crop can only build a larger population in days when the flowers blossom. In contrast, the other group has several growing points.

6.2 Egg Laying Rate

We adjust the egg laying rate of the queen. From figure 6.2, the more eggs the queen can lay, the bigger the population is. That is because more bees can be born in times with plenty of food and eggs. Thus, a queen laying 3,000 eggs per day can make a colony much larger than a colony with a queen producing less eggs.

6.3 Fertilize Rate

We adjust the fertilize rate and find out that the higher the fertilize rate is, the larger the population is. A higher fertilize rate means more workers are born. With more workers, the broods can easily

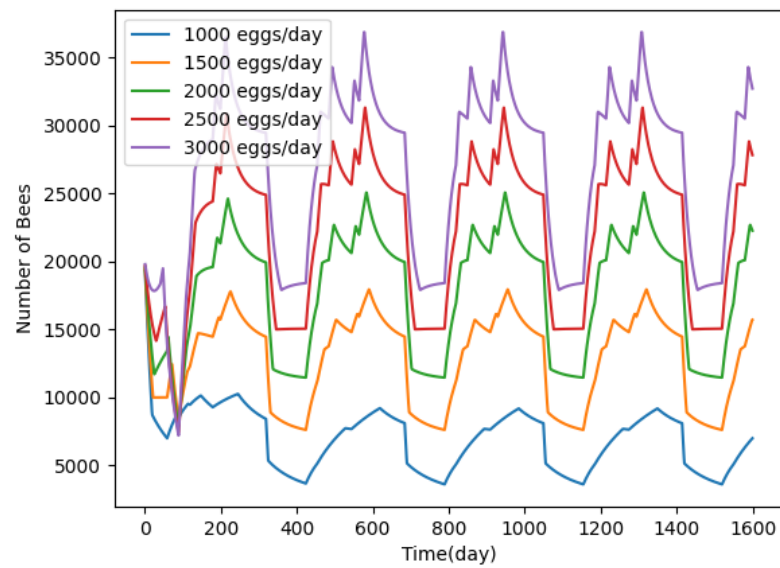


Figure 6.2:

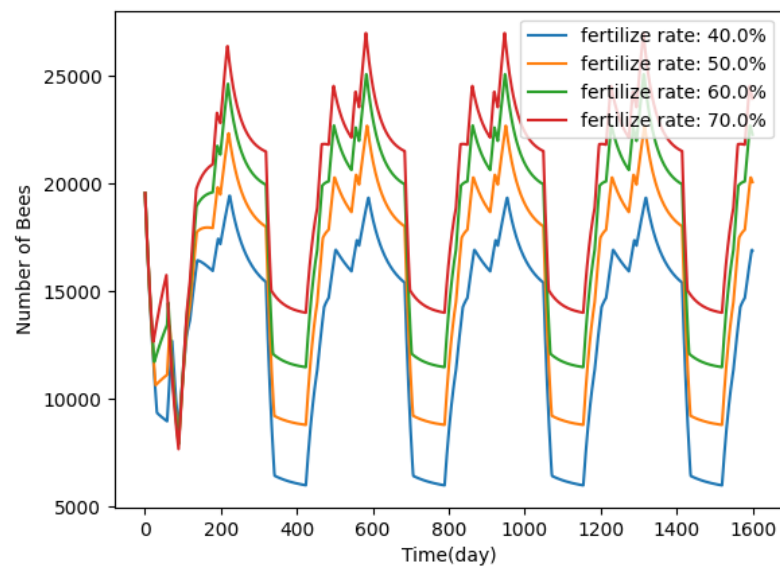


Figure 6.3:

live and increase the total population. In contrast, if more drones are born, there won't be enough workers to care for the broods.

6.4 Lifespan

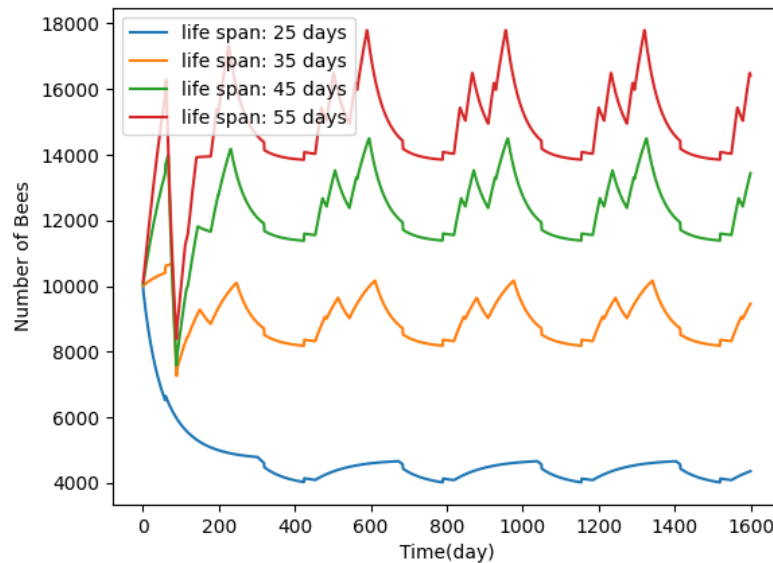


Figure 6.4:

After changing the lifespan of all bees, we see that a short life span can greatly restrain the colony's population. Colonies with bees living longer lives have hundreds of bees more than those with bees living shorter lives.

6.5 Pesticide

By adding each of the pesticides to our model, we find out that the bee population will decline greatly even if the pesticides are used once a month. The pesticide with a higher fatality rate can easily make the population drop to zero. That is because the birth rate and food can't support the death rate. Thus, pesticides are fatal to many bee colonies. Using less pesticides is protecting more bees.

7 Hive Number Prediction

The following model aims to provide a method to estimate the number of honey bee hives required to support pollination of a 20-acre (81,000 square meters) parcel of land planted with crops relying on bee pollination. We will mainly focus on the pollination of crops previously discussed in section 4.4, which are Alfalfa, Date Palm, Summer Squash, as well as Sunflower. Also, we will assume that the shape of the crop field is rectangular, and the dimensions of the field will be discussed in the next paragraph.

We now show that due to the given area of the land, it is unlikely for bees to bypass the 6km distance limit from their hives. Consider a rectangular field with diagonal length 6km and area

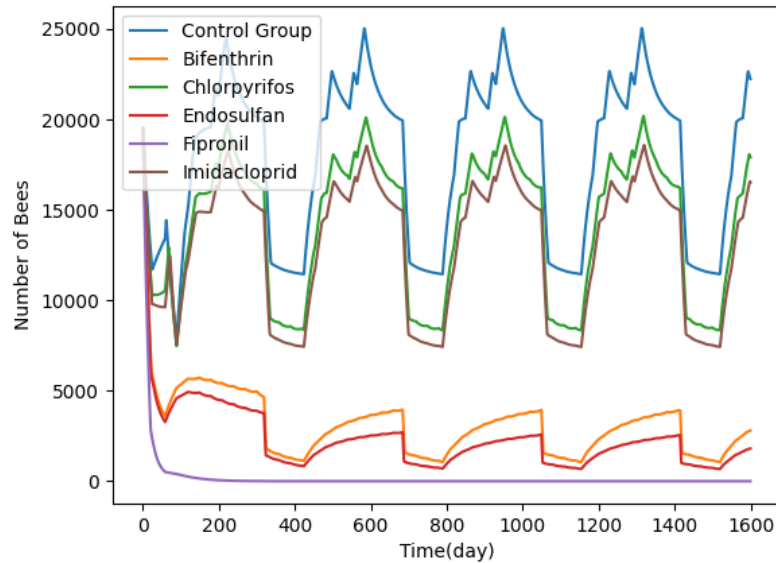


Figure 6.5:

$81,000m^2$. We consider this extreme case since the diagonal distance is the longest distance in a rectangle. Using the Pythagorean Theorem, we could calculate that a rectangle with hypotenuse 6000m and area $81,000m^2$ has side lengths approximately equal to 13.5 meters and 6000.0 meters. The following graph shows an evenly-downsized shape of a rectangle with dimensions given above.



Figure 7.1:

Please keep in mind that the longer length of the rectangle will increase as the diagonal length exceeds 6km. Therefore, Such idiosyncrasy can be neglected in our discussion.

Next, figure 7.2 shows the graph of u (the number of bees emerging from a hive per minute) to T (temperature measured in degrees Celsius). The temperature ranges from 14 degrees celcius to

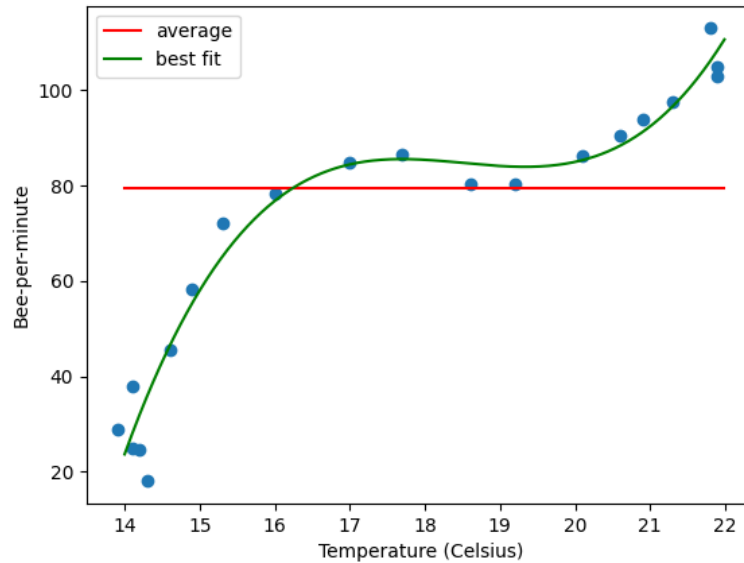


Figure 7.2:

22 degrees celcius. The data set is obtained from Robert M. BURRILL and Alfred DIETZ's study in 1981. We fit the data set into a cubic function defined as follows:

$$u = f(T) = 0.738290558T^3 - 40.001211T^2 + 757.518838T - 4571.24258$$

We then calculate the mean value (Mv) of this continuous function (shown as the red line in Figure 7.2):

$$Mv = \frac{\int_{14}^{22} f(T)dT}{22 - 14} = 79.3692$$

At a given temperature T_0 , $f(T_0)$ represents the number of bees, namely foragers, emerging from the hive per minute. When we divide $f(T_0)$ by Mv , the relationship between the two will be obtained, and we call the resulting rate R .

$$R = \frac{f(T_0)}{Mv}$$

For example, when the temperature is 15 degrees, the number of bees emerging per minute is approximately 60. Thus R approximates as 0.76. This number shows that $f(15)$ reaches 76 percent of the average Mv . To be more general, 76 percent of the predicted number of working bees will pollinate the crops at $T=15$. However, as $f(T)$ and the average line intersects at $T=16.2$, the number of working bees equals and never exceeds the predicted amount.

Now we move on to analyzing the four crops and their planted-place distribution. "Place" in the table stands for the location where such crop is commonly-found.

Crop	Place	Temperature (degree Celsius)	Pollinating Season
Alfalfa	California	21.4	Summer
Date Palm	California	20.6	Late Summer
Summer Squash	California	21.4	Summer
Sunflower	Moscow	14.1	Summer to Autumn

For further calculations, we also list out the data of crop density in the following table. Please note that for the sunflower column, we are considering its seeds amount, since a single sunflower consists of around 1,000 seeds.

Crop	Flower/acre
Alfalfa	10 million
Date Palm	10 thousand
Summer Squash	25 thousand
Sunflower	21 million

Now, we are ready to calculate the number of hives needed to fulfill pollination needs for each of the crops listed above. Denote the number of Alfalfa flowers, Date Palms, Summer Squashes, as well as Sunflower Seeds per acre as $S_a, S_d, S_{s_1}, S_{s_2}$, respectively. Also, denote the number of bees needed for pollination as $B_a, B_d, B_{s_1}, B_{s_2}$. Let the respective temperatures where Alfalfa flowers, Date Palms, Summer Squashes, and Sunflower Seeds grow be $T_a, T_d, T_{s_1}, T_{s_2}$. Further, let the number of hives needed to pollinate a 20-acre Alfalfa, Date Palm, Summer Squash, and Sunflower land be $q_a, q_d, q_{s_1}, q_{s_2}$. Recall that $W(t)$ represents the function of working bee population to time. We can transform "seasons" into t (time in days).

Take Alfalfa flowers as an example. Assume that a single honey bee visits 2,000 flowers in a single day. thus

$$B_a = \frac{S_a}{2000} = 5000$$

Since T_a lies within (16.2, 22), the number of foraging working bees equals the predicted amount $W(t)$ at $t = 220$. Thus using our prediction model, we get $W(220) = 10000$. Taking the ceiling of $20 * \frac{B_a}{W(220)}$ concludes that $q_a = 20$

Therefore we will need 20 hives to pollinate a 20-acre land planted with Alfalfa flowers. Similar results can be drawn to Date Palms and Summer Squashes, while Sunflowers require special consideration due to Moscow's temperature.

$$\text{Observe that } R = \frac{f(14.1)}{M_v} \approx 0.31, \text{ and } W(380) = 12000$$

We then take the ceiling of $\frac{W(380)*R}{S_{s_2}/2000}$, and multiply it by 20. This gives us a need of 60 hives in total. The chart below shows our answer to the required number of hives:

q_a	20
q_d	20
q_{s_1}	20
q_{s_2}	60

8 Strengths and Weaknesses

8.1 Strengths

- **Relatively accurate**

Despite the limited number of variables in our model compared to models such as BEEHAVE, the model results strongly resembles that of actual bee colonies [2, 13]

- **Identify pesticide as one of the primary causes of colony collapse disorder**

Our model identifies pesticide as one of the primary causes of colony collapse disorder, which corresponds to the findings of other papers [14]. Using this information, we know that limiting pesticide usage is indeed an important way of protecting the bees.

8.2 Weaknesses

Our model doesn't take into account the effects of habitat [5, 6] loss, predators, and other factors that might affect the population of honey bees because these factors vary case to case and is hard to express with a generalized mathematical model.

9 Conclusion and discussion

Our population model shows that while a lack of crop variety, a relatively low egg laying rate and a low fertilization cause the colony's population decline, the colony's population was steady over the four years. When the background seasonal mortality rate of bees increased to above $\frac{1}{30}$, or when strong insecticides such as fipronil was used, colony collapse disorder was observed. Thus, we should limit the use of strong insecticides to protect the bees.

Our hive number prediction model shows that approximately 20 hives are required to pollenate 20 acre of crops. However, when the crop density is high and the climate condition in that region is harsh, approximately 60 hives are needed.

Admittedly, adding other factors such as habitat loss into the model would make it more comprehensive. However, our model can accurately depict the daily fluctuations of bee population over the course of four years.

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10 Code

```

import numpy as np
from matplotlib import pyplot as plt

pesticide_data = [("Control Group",1,1),
                  ("Bifenthrin",87,0.015),
                  ("Chlorpyrifos",50,0.072),
                  ("Endosulfan",86,0.014),
                  ("Fipronil",142,0.007),
                  ("Imidacloprid",174,0.061)]

def dose_function(time,half_life):
    = np.log(2)/half_life #np.log is ln
    dose = 0.001*np.e**(- *time)
    return dose

def mortality(time,half_life,LD):
    k = np.log(3)/LD
    dose = dose_function(time,half_life)
    m = 2/(1+(np.e**(-k*dose))-1)
    return m

def add(a,b):
    return [a[i]+b[i] for i in range(len(a))]

def sub(a,b):
    return [a[i]-b[i] for i in range(len(a))]

def mult(a,b):
    return [i*b for i in a]

def protein(A,D,SS,S,R):
    global worker,npf
    bee_population = worker
    acid_percentage = (
        (1.55,1.71,3.56,6.38,3.15,0.22,1.51,2.34,0.60,4.92),
        (2.12,1.62,3.30,6.14,4.51,0.18,1.53,2.45,0.61,4.61),
        (1.75,3.83,3.54,7.14,4.68,0.43,1.57,2.44,0.63,5.23),
        (2.56,4.00,3.04,6.35,4.98,0.24,0.76,3.41,0.33,5.46))
    requirements = [3.00,1.50,4.00,4.50,3.00,1.50,1.50,1.50,1.00,4.00]
    pA = A*npf*20.23/100
    pD = D*npf*19.77/100
    pSS = SS*npf*16.39/100
    pS = S*npf*15.19/100
    pR = R*npf*18.86/100
    kinds = [[pA*acid_percentage[0][i]/100 for i in range(10)],
             [pD*acid_percentage[1][i]/100 for i in range(10)],
             [pSS*acid_percentage[2][i]/100 for i in range(10)],
             [pS*acid_percentage[3][i]/100 for i in range(10)]]
    total = [(kinds[0][i]+kinds[1][i]+kinds[2][i]+kinds[3][i]) for i in range(10)]
    required_acids = [bee_population*requirements[i]/100 for i in range(10)]
    return total

def get_food_source(time): # gain food in a specific day
    global A,D,SS,S,worker,drone
    required_acids = [3.00,1.50,4.00,4.50,3.00,1.50,1.50,1.50,1.00,4.00] #mg
    required_acids = [i*0.001 for i in required_acids]
    ps = [0,0,0,0,0,0,0,0,0,0] #protein_storage
    #time += 300

```

```

    if time % 365 >= 120 and time % 365 < 127:
        ps = add(ps,mult(protein(A,0,0,0,0),1/7))
    if time % 365 >= 90 and time % 365 < 97:
        ps = add(ps,mult(protein(0,D,0,0,0),1/7))
    if time % 365 >= 200 and time % 365 < 207:
        ps = add(ps,mult(protein(0,0,SS,0,0),1/7))
    if time % 365 >= 180 and time % 365 < 187:
        ps = add(ps,mult(protein(0,0,0,S,0),1/7))
    ps = sub(ps,[i*int(worker+drone) for i in required_acids])
    #for i in range(len(ps)):
    #    if ps[i] <= 0:
    #        ps[i] = 0
    return ps

def death_of_hunger():
    required_acids = [3.00,1.50,4.00,4.50,3.00,1.50,1.50,1.50,1.00,4.00] #mg
    required_acids = [i*0.001 for i in required_acids]

    global worker,drone,food_source
    #print(food_source,[food_source[i]/required_acids[i] for i in range(len(food_source))])
    death = -round(min([food_source[i]/required_acids[i] for i in range(len(food_source))]))
    if death > 0:
        #print(death)
        #print(food_source)
        food_source = add(food_source, [required_acids[i]*death for i in range(len(required_acids))])
        #print(food_source)
        worker -= worker/10000*death/200
        drone -= drone/10000*death/200

def natural_death():
    global day,worker,drone
    if day % 365 >= 320 or day % 365 < 60:
        worker *= 59/60
        drone *= 59/60
    else:
        worker *= 44/45
        drone *= 59/60

def pesticide_death(pesticide_mortality):
    global worker
    worker *= (1-pesticide_mortality)

time = 1600 # days
egg_rate = 2000 # per day
fert_rate = 0.6 # of the eggs are female
#hatching_rate = 0.75 # of the eggs can successfully hatch
npf = 10 #nectar per flower
food_source = [3000.0, 1500.0, 4000.0, 4500.0, 3000.0, 1500.0, 1500.0, 1500.0, 1000.0, 4000.0]
A = 100000 #
D = 100000
SS = 100000
S = 100000

worker = 10000 # how many workers in colony
drone = 10000

```

```

out = []
out2 = []
out3 = []
out4 = []
for day in range(1,time):
    food_source = add(get_food_source(day),food_source)
    food_source = [i*0.995 for i in food_source]
    death_of_hunger()
    natural_death()
    #if day >= 100:
    #    pesticide_death(mortality(day,pesticide[1],pesticide[2]))
    #else:
    #pesticide_death(mortality(day%100000,pesticide[1],pesticide[2]))
    total_birth = egg_rate * np.prod(food_source) / (np.prod(food_source)+20**5) * worker / (w
    worker_birth = fert_rate * total_birth
    drone_birth = (1-fert_rate) * total_birth
    death_rate = 0
    worker += worker_birth
    drone += drone_birth
    if day % 365 >= 320 or day % 365 < 60:
        drone -= 500
        if drone <= 0:
            drone = 0
    out.append(worker)
    out2.append(drone)
    out3.append(food_source)
    out4.append(worker+drone)

#plt.plot(range(1,time), out4)
plt.plot(range(1,time), out, label = "Worker")
plt.plot(range(1,time), out2, label = "Drone")
plt.ylabel("Number of Bees")
plt.xlabel("Time(day)")
for j in range(10):
    #plt.plot(range(1,time), [i[j] for i in out3])
    #plt.ylabel("Amount of Acids")
    pass
plt.legend()
plt.show()

```

11 simplified process map

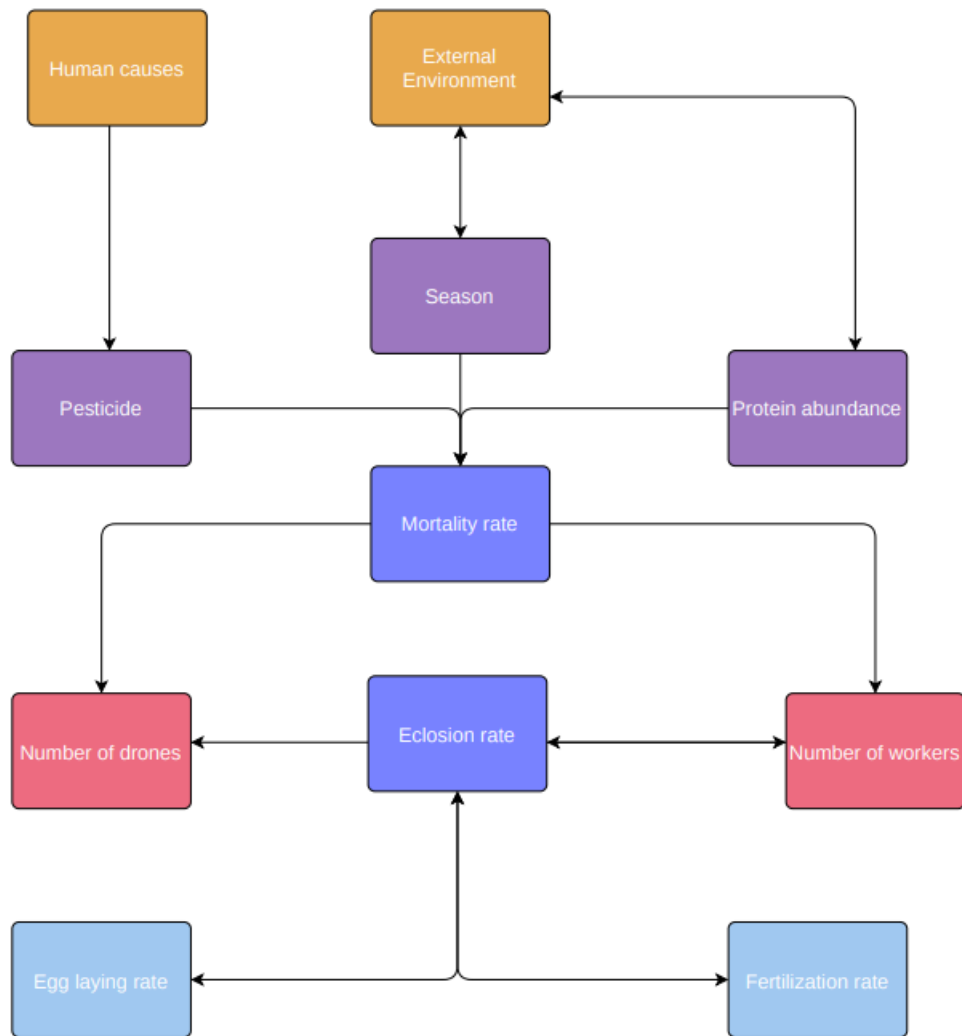


Figure 11.1: