Team Control Number

14450

Problem Chosen

A

2023

HiMCM

Summary Sheet

A Detailed Model for Dandelion Spread Prediction Based on Wind Roses

Dandelions (*Taraxacum officinale*) are famous for their distinctive means of seed dispersal: **wind dispersal**. Their characteristic parachute structures enable them to travel long distances by wind and thus increase the chance of survival of their seeds. Thanks to this trait, they are classified as potentially invasive species since they can colonize new areas quickly. Our study aims to design a model to predict the spread and invasiveness of dandelions.

To begin, we propose several assumptions and carry out their justification to simplify the research problem. Next, various factors are incorporated into the model, including **wind effect, precipitation, temperature, growth rate, and survival rate**. With reference to findings from previous research, we calculate the dispersal distances at different wind speeds and apply **wind roses** to determine the probabilities of pappus landing in various spots. We use an innovative approach, the **progress bar method**, to calculate the expected time for a dandelion to reach maturity while considering multiple environmental factors and the effect of intraspecies competition. Lastly, the model takes into account of mortality by stimulating a function that relates survival rate to age, precipitation, and intraspecies competition. As a result, the model accurately predicts the spread and total population of dandelions over time.

In addition, we conduct a sensitivity analysis to identify the impact of each factor and prove the stability and validity of our model. The parameters include wind roses at different locations, wait time, implantation date, precipitation, initial survival rate, and competition severeness. We analyze the impact of each factor by systematically changing their values while keeping other variables constant (*ceteris paribus*). Ultimately, we conclude that **the severity of intraspecies competition** plays an integral role in the determination of long-term dandelion spread.

As we progress in our research, we use **TOPSIS** (Technique for Order of Preference by Similarity to Ideal Solution) to quantify an impact factor for the invasiveness of dandelions. By creating a data set containing different plant species and various criteria related to invasiveness, we determine the impact factors for each selected plant. As a result, we conclude that dandelion species possess a **relatively high invasiveness**. To confirm the validity of our result, we test our model with two other species known for their invasiveness, and their impact factors are indeed high, proving the validity of our invasiveness model.

Key Words: Dispersal Model; Population Model; Wind Roses; Seasonality; Intraspecies Competition; *Taraxacum Officinale*

Contents

1 Introduction

1.1 Background

Dandelion (*Taraxacum Officinale*), a commonly-seen plant, is renowned for its unique appearance and special means of seed dispersal. After pollination, the dandelion will transform from yellow flowers to puffballs composed of parachute-like structures. Under these fluffy parachute-like structures lie its seeds. With the help of the unique structure and optimal wind conditions, dandelion seeds can travel far distances theoretically up to 100 kilometers [1]. Normally, dandelions disperse over much shorter distances and germinate where they land.

Dandelions' dispersal traits allow them to spread and colonize new areas rapidly and are therefore sometimes considered as weeds and invasive species. In areas with large amounts of dandelions, competition among plants for resources and space intensifies, negatively impacting the native plants. Plus, Kandori et al's research in 2009 [2] reveals that dandelions can reduce "the reproduction of a native congener through competition for pollination." Thus, dandelion's rapid and extensive spread can trigger severe problems.

However, dandelions also contribute plentiful economic and societal benefits. Their flowers serve as a vital food source for pollinators, aiding in the pollination of other plants. Their deep roots can not only improve the soil quality, but also prevent soil erosion. Dandelions' culinary and medical use are also of integral importance.

Therefore, dandelions have a complicated relationship with the environment and humans, so studying the spread of such species and their potential impacts to the environment becomes indispensable.

1.2 Problem Restatement

Our research focuses on the determination of dandelion dispersal over a certain time interval and the impact factor of dandelion on the environment. We will provide two different models to solve each problem.

- A dispersal model should first be constructed to describe pappus spreading and dandelion growth over a certain month. The variables that could affect the spreading of dandelion pappus and growth such as wind speed, wind direction, and temperature should be included in the model.
- Second, we need to conduct a sensitivity analysis on the population model to see how much each factor affects pappus dispersal. We need to alter each factor while holding the others constant and then quantitatively examine how it impacts the spreading of dandelion pappus. The factors can then be ranked according to their impact.
- Third, we should apply the model to estimate the spread of dandelions on an open one-hectare plot of land over the course of 1, 2, 3, 6, and 12 months.
- Then, we need to establish a model to determine the impact of dandelions on the environment.
- Last but not least, we should test our second model by determining the impact factor for two predetermined invasive plant species.

1.3 Assumptions and Justifications

- **Assumption 1:** We consider the dandelion spread to be discrete instead of continuous.

  Justification: Since we are interested in time intervals of at least one month, continuous spreading and growth are not necessarily required.

- **Assumption 2:** Disregarding geographic factors, wind is the only factor influencing the spreading distance, while rainfall affects the number of dandelion offspring.

  Justification: Dandelions primarily rely on wind dispersal. Precipitation incites the growth of seeds before dispersal stage, but inhibits the spread of mature pappus.

- **Assumption 3:** There is no artificial effect.

  Justification: Due to the complexities of artificial effects, only natural factors are taken into account.

- **Assumption 4:** Different dandelions are considered identical in pappus size and shape.

  Justification: Accounting for the variations in size and shape would introduce additional complexities as these factors can impact the seeds' dispersal abilities. We can simplify the modeling process by assuming such uniformity.

- **Assumption 5:** There are 30 days in each month.

  Justification: Dandelions have quite a long growing period, so we unify the number of days in a month for simplicity.

2 Modeling the Spread of Dandelions

2.1 Variables

Variable Symbol	Meaning
A_d	age of dandelion
$A_{thriving}$	a dandelion's thriving age
c_c	intraspecific competition coefficient
d_p	distance the pappus traveled
d_i	distance between dandelions
f_{v_i,θ_j}	frequency of wind at strength v_i and direction θ_j
F_r	rainfall frequency per month
h_p	height of the dandelion
l_p	dandelions' pappus length
m_p	mass of dandelion pappus
n	number of dandelions
P_{wet}	probability of pappus detachment in wet conditions
P_{dry}	probability of pappus detachment in dry conditions
P_{v_i,θ_j}	probability that the wind blows at strength v_i and direction θ_j
P_{d_p,θ_j}	probability that the pappus travels a distance d_p and lands in direction θ_i
q	progress of dandelion maturity
r_T	temperature factor on maturity rate
r_c	intraspecies competition factor on maturity rate
r_e	environmental factor on maturity rate
R_d	Radius of a dandelion
R_s	Radius parameter when assessing survival rate
t_s	time the pappus stays in the air
v_p	vertical pappus speed
w	wait time

Table 1: Variables used in the model.

2.2 Wind Effect

2.2.1 Wind Characteristics

Wind is the most important factor when analyzing the spread of dandelions, and it is commonly measured by two different parameters: speed v_w and direction θ. *Wind rose diagram* is a clear and accessible visualization of the distribution of wind at a certain speed and direction. A wind rose can quickly indicate the wind frequencies at different strengths and directions, offering vital information for our model. Iowa Environment Mesonet provides statistics for wind direction and speed with raw data and wind roses differing by month [3]. Our spreading model depends largely on the data obtained from wind roses at various places and time periods. By using the information derived from the wind roses, our model can provide an accurate prediction of the spread.

For example, in the wind rose demonstrated in fig. 1, the angular axis is the direction of the wind θ; the radial axis is the frequency of the wind blowing in a certain direction or distance; the color represents the wind speed.

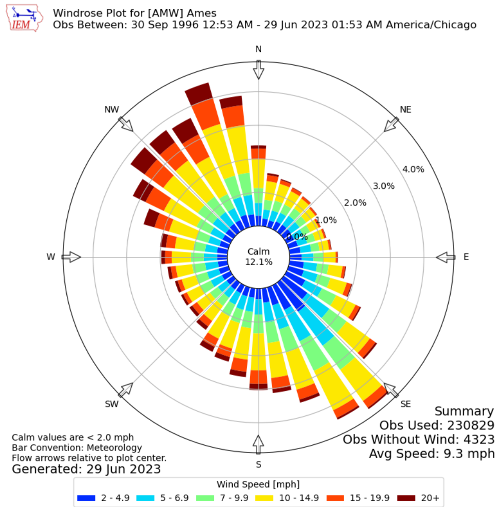


Figure 1: A yearly wind rose plot for Ames, America.

2.2.2 Mapping Wind Speed to Pappus Dispersal Distance

We assume that there is a direct correspondence between wind strength and dispersal distance. That is, there is a one-one function mapping f_{θ,v_w} to f_{θ,d_p}.

The model provided by [4] suggests that pappus mass m_p and pappus hair length l_p determine the dispersal potential of dandelion pappus. This model suggests a linear relation between pappus' vertical falling speed, pappus mass m_p, and pappus hair length l_p:

$$v_p = b_0 + b_1 \cdot m_p + b_2 \cdot l_p, \tag{2.1}$$

where b_0, b_1, b_2 are coefficients.

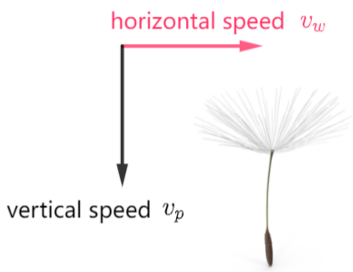


Figure 2: We assume that the wind flows horizontally, and the falling speed v_p will affect the distance traveled d_p.

We separate the pappus motion into the vertical and horizontal directions. The pappus settles on the ground after traveling a distance of h_p vertically with an average fixed speed v_p. So the pappus stays in the air for a time interval $t_s = h_p/v_p$. At this time interval, the horizontal distance traveled is given by

$$d_p = v_w \cdot t_s = \frac{v_w \cdot h_p}{b_0 + b_1 \cdot m_p + b_2 \cdot l_p}, \tag{2.2}$$

where $b_0 = 0.674$, $b_1 = 0.394$, $b_2 = -0.081$ as supported by the linear regression model proven by the data in [4].

In this model, we assume that $h_p = 0.3$ m, $m_p = 4 \times 10^{-7}$ kg, $l_p = 6 \times 10^{-3}$ m.

2.2.3 Mapping Pappus Dispersal Distance to Probability

The wind roses' data accounts for a whole month, but the wind at one specific time has a fixed direction and strength. Since dandelions do not disperse all their seeds at once, their seeds are going to fly to different places according to the changing wind. Therefore, we must employ a probabilistic model to decide where the seeds land. The probability is determined by the data from the wind roses.

The raw data from the wind roses can be shown in this table, with f_{v_i,θ_j} representing the frequency of wind at strength v_i and direction θ:

Direction (°)	Calm	Wind Speed (m/s)					
		2.0-4.9	5.0-6.9	7.0-9.9	10.0-14.9	15.0-19.9	20.0+
355-004	f_{v0,θ_1}	f_{v1,θ_1}	f_{v2,θ_1}	f_{v3,θ_1}	f_{v4,θ_1}	f_{v5,θ_1}	f_{v6,θ_1}
005-014	f_{v0,θ_2}	f_{v1,θ_2}	f_{v2,θ_2}	f_{v3,θ_2}	f_{v4,θ_2}	f_{v5,θ_2}	f_{v6,θ_2}
⋮							
335-344	$f_{v0,\theta_{35}}$	$f_{v1,\theta_{35}}$	$f_{v2,\theta_{35}}$	$f_{v3,\theta_{35}}$	$f_{v4,\theta_{35}}$	$f_{v5,\theta_{35}}$	$f_{v6,\theta_{35}}$
345-354	$f_{v0,\theta_{36}}$	$f_{v1,\theta_{36}}$	$f_{v2,\theta_{36}}$	$f_{v3,\theta_{36}}$	$f_{v4,\theta_{36}}$	$f_{v5,\theta_{36}}$	$f_{v6,\theta_{36}}$

In this table,

$$\sum_{i=0}^{6}\sum_{j=1}^{36} f_{v_i,\theta_j} = 1. \tag{2.3}$$

To begin, the probability for a seed to land in direction θ_i is determined by the wind frequency at this direction, namely, the length of the color bar at direction θ_i in the wind roses:

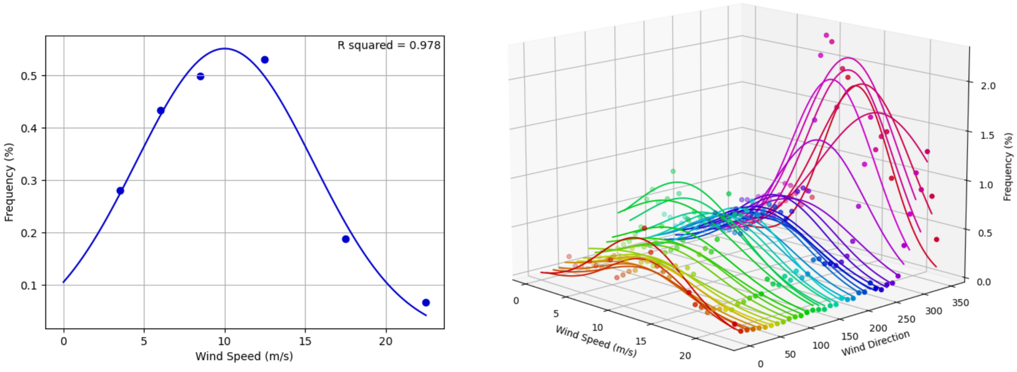
$$P_{\theta_j} = \sum_{i=1}^{6} f_{v_i,\theta_j}. \tag{2.4}$$

Then, we apply curve fitting to calculate the probability that a seed spreads a specific distance in a fixed direction. By applying curve fitting techniques, we can determine the mathematical relationship between the distance traveled by the seed and the corresponding probability. This allows us to estimate the likelihood of seeds spreading various distances based on the available data.

We approximate the wind speed as the median of each interval. For example, we assume the wind speed is 2.5 m/s at interval 2.0-4.9 m/s. Plus, since the directions contain 36 intervals but not accurate angles, we randomly select an angle in each interval for the seed to spread. By utlizing the method of mapping wind speed to pappus dispersal distance as discussed above, we get a chart demonstrating respective frequencies of certain wind speed and directions:

v_w (m/s)	d_p (m)	f_{v_i,θ_j}
0	d_{p0}	f_{v0,θ_j}
2.5	d_{p1}	f_{v1,θ_j}
6.0	d_{p2}	f_{v2,θ_j}
8.0	d_{p3}	f_{v3,θ_j}
12.5	d_{p4}	f_{v4,θ_j}
17.5	d_{p5}	f_{v5,θ_j}
22.5	d_{p6}	f_{v6,θ_j}

Take Ames's windrose as an example; we observe that the relation between frequency and wind speed is similar to that of normal distributions, so we apply normal distribution curves to fit these curves. It turns out that they fit well, with high R^2 around 0.9. The relationships $f(v_w)$[1] at different directions are shown in these diagrams:



(a) $f(v_w)$ in one direction (b) $f(v_w)$ in all 36 directions

Figure 3: Curve fitting of frequency against wind speed

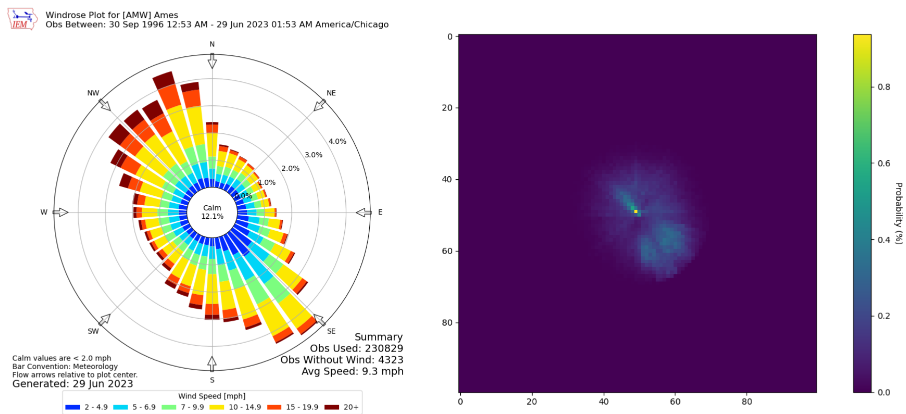
By combining $f(v_w)$ with the equation calculating d_p, we obtain the function $f(d_p)$. To calculate the probability for a seed to travel a certain distance, we use the formula

$$P_{v_i} = \frac{f(d_p)}{\displaystyle\int_0^{d_{p6}} f(d_p)\,d(d_p)}. \tag{2.5}$$

Therefore the probability for a seed to land at one spot is calculated by

$$P_{v_i,\theta_j} = P_{v_i} \cdot P_{\theta_j}. \tag{2.6}$$

For example, the respective landing probabilities for seeds are shown in the following diagram, where the wind rose comes from Ames in January. The color depth at a certain point represents the probability of landing at that point.



(a) (b)

Figure 4: Wind Rose and Probabilities Diagram

[1]Here we use the notation $f(v_w)$ because it is a continuous variation, so does $f(d_p)$.

2.3 Rainfall Effect

Dandelion pappi are more likely to attach to their capitulum when wet. Moist pappi are caused by rainfall. We hypothesize that the presence of rainfall (regardless of the amount of precipitation) affects the humidity of the pappi, resulting in less pappus spreading.

2.3.1 Rainfall to Humidity

During rainfall, we assume that the likelihood of spreading is 0 because the pappi will be carried directly to the ground by the rain. After rainfall, there is a period during which the dandelion pappus remains wet, and therefore less likely to spread. Afterward, the dandelion pappus will be completely dry and able to spread successfully. The time is assumed to be 1 day and the monthly frequency of rainfall F_r can be calculated as

$$F_r = \text{rainy days/total days in a month.} \tag{2.7}$$

Take Ames's rainfall as an example, we have F_r for different months listed below.

Month	$F_r(\%)$	Month	$F_r(\%)$	Month	$F_r(\%)$	Month	$F_r(\%)$
January	21.0	February	27.5	March	30.5	April	40.3
May	40.3	June	44.0	July	33.8	August	33.8
September	32.5	October	30.3	November	20.2	December	30.4

2.3.2 Humidity to Detachment Percentage

The correspondence between humidity and detachment is shown in [5]. Suppose that the percentage of detachment in dry and wet environments are P_{dry} and P_{wet}, respectively. We claim that there exists a correspondence between P_{dry} and P_{wet}.

$$P_{wet} = C \cdot P_{dry}, \tag{2.8}$$

where C is a constant deduced by the data given in [5]. Our calculation suggests that the coefficient $C \approx 0.529$.

Therefore, the total possibility of detachment is

$$P_{detach} = F_r P_{wet} + (1 - F_r) P_{dry}. \tag{2.9}$$

2.4 Wait Time Adjustment

Since dandelions do not immediately turn into puffballs after their mother plant's dispersal, a wait time variable accounting for the duration between their planted date and their scattering stage is required. In this section, we will discuss the determination of the wait time w for dandelions dispersed at a specific date. We use $q \in [0, 100]$ to account for a single dandelion's maturity rate. Alternatively, q can be understood as a progress bar measuring a dandelion's maturity: the closer q is to 100, the closer the dandelion being measured is to the dispersal stage. The wait time is equivalent to the time it takes for the maturity rate to grow from 0 to 100.

We have isolated three major factors to account for the growth in maturity rate: r_T, r_c, and r_e.

$$q = r_T \cdot r_c \cdot r_e. \tag{2.10}$$

We impose a normal distribution function on r_T, the rate of change of q (assuming that r_c and r_e are temporarily equal to 1) to the number of days, and utilize the integral of the normal distribution function to find the wait time for plants planted at a specific date. The utilization of such a model could be justified by the normal distribution relation between plant growth and time of year, [6]. This model effectively incorporates the effect of temperature on growth rate by considering the time of year when particular seeds are implanted.

For r_c, we consider the effect of intraspecific competition on dandelion growth. We primarily consider the effect of nearby plants on the dandelion.

For r_e, we consider the effect of the environment on dandelion growth; a precipitation factor is of major concern.

2.4.1 Temperature Factors

We use $q(t)$ to denote the function of the change in maturity rate to time (measured in days) and assume that this function has a period of 360 days. If we further assume that dandelions reach a maximal and minimal maturity speed in June (180 days) and January (0 days), respectively, [7] we could derive the coefficients for

$$q(t) = \frac{h}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{t-\mu}{\sigma})^2}. \tag{2.11}$$

Using our assumption that $q(0) = v_{min}$ and $q(180) = v_{max}$, we deduce that

$$\begin{cases} \mu = 180, \\ \sigma = \sqrt{\dfrac{\mu^2}{2\ln\left(\frac{v_{max}}{v_{min}}\right)}}, \\ h = \mu v_{max}\sqrt{\dfrac{\pi}{\ln\left(\frac{v_{max}}{v_{min}}\right)}}. \end{cases} \tag{2.12}$$

To find the wait time (Δt) for dandelions planted at time t, we make use of $\Phi(t)$, the cumulative distribution function of normal distributions, given as

$$\Phi(t) = \frac{1}{2}\int_{-\infty}^{t} e^{-\frac{u^2}{2}}\, du. \tag{2.13}$$

Since we need to find the time for the maturity rate to shift from 0 to 100, we need to solve for Δt in the following equation.

$$\int_{t}^{t+\Delta t} q(t)\, dt = 100. \tag{2.14}$$

Equivalently,

$$\Phi\left(\frac{t+\Delta t-\mu}{\sigma}\right) - \Phi\left(\frac{t-\mu}{\sigma}\right) = \frac{100}{h}. \tag{2.15}$$

Although $\Phi(t)$ cannot be expressed in elementary form, plenty of numerical values have been assigned to $\Phi(t)$, allowing possibilities for computation. Thus, we directly derive an equation for r_T

$$r_T = \Delta t = \sigma\Phi^{-1}\left(\Phi\left(\frac{t-\mu}{\sigma}\right) + \frac{100}{h}\right) + \mu - t. \tag{2.16}$$

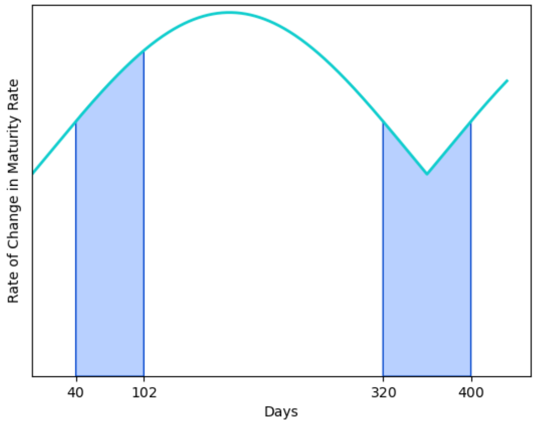


Figure 5: Demonstrating the determination of Δt.

2.4.2 Intraspecies Competition

Dandelions dispersed near each other compete for the same resources and thus have a limiting factor on particular dandelions' maturity rate. We introduce a factor r_c to quantify the influence posted by nearby dandelions on a particular dandelion. We assume that all available resources are evenly distributed among the dandelions within our consideration. We propose the following function for r_c

$$r_c = \frac{1}{\sum_{i=1}^{n}(R_d - d_i) + 1}, \tag{2.17}$$

where n denotes the number of dandelions within circular distance R of the dandelion we are considering.

Taking into consideration that dandelion roots have an average radius of 2 centimeter and dandelions' root radii affect its nutrient uptake, [8] R is chosen to be between 2cm and 30cm in length. [9] A sensitivity analysis was made on the choice of R. As the sum increases, the average distance to the dandelion decreases. As a result, r_c decreases, causing a drop in maturity rate.

The number of nearby dandelions (n) is calibrated after each dispersal according to fig. 6.

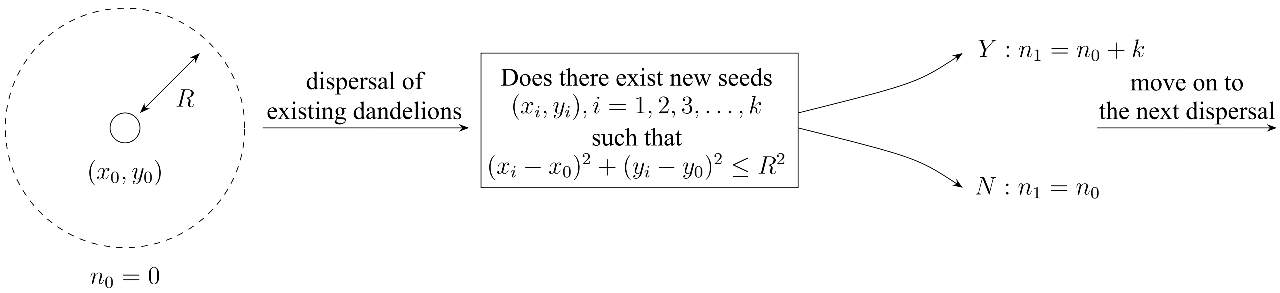


Figure 6: Flow chart for the determination the number of seeds within R cm of the dandelions.

2.4.3 Environmental Factors

The key environmental factor affecting maturity rate growth is precipitation, and we employ the factor F_r to account for the growth of maturity rate attributed to precipitation. [10] There should be a positive correlation between F_r and r_e. We let

$$r_e = 1 + (F_r - m_0)^{m_1}. \tag{2.18}$$

We use m_0 as a threshold value for *effective* rainfall: if F_r exceeds m_0, the precipitation amount is considered favorably excessive, and vice versa. A power m_1 is used to diminish the effect caused by rain. We calibrate m_0 to be 0.30, and m_1 to be $\frac{1}{31}$.

2.4.4 Robustness

A random variable within $[0.9, 1.1]$ is assigned to q to account for the difference between dandelion robustness. For each dispersed seed, such a variable accounts for the observable difference between individual dandelions in reality.

2.5 Survival Rate Model

Several factors such as bad weather and caterpillar infection inhibit the germination of dandelion seeds. Thus, we need to employ a survival rate to account for deaths caused by such factors.

In real-world conditions, survival rate is low for relatively young and old dandelions. The curve of survival rate against age should grow gradually at first and decrease dramatically after a certain age. This can be proven by considering human survival rates at different ages [11]. Although human's living condition is quite different, the overall trend should be the same. However, newborn plant's survival rate should be lower than that of human.

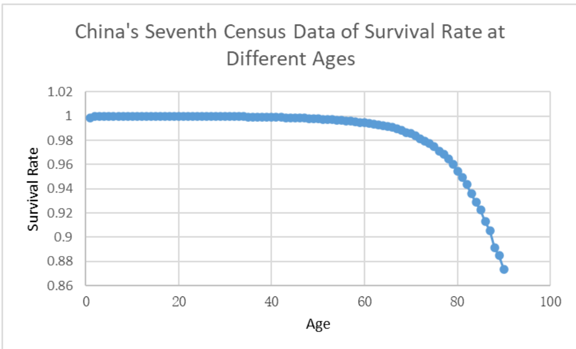


Figure 7: Survival rate against age diagram in China

We design the following formula to satisfy the characteristics discussed above, where P is the surviving possibility for an individual and A is a dandelion's age.

$$\begin{cases} P_{survival} = [k_1(A - k_2) + \dfrac{1}{A - k_2} + k_3] \cdot k_s, \\ 2\sqrt{k_1} + k_3 = 1, \\ -k_1 \cdot k_2 - \dfrac{1}{k_2} + k_3 = P_{initial}, \\ \dfrac{\sqrt{k_1}}{k_1} + k_2 = A_{thriving}, \\ k_s = \dfrac{c_c}{\sum_{i=1}^{n}(R_s - d_i) + c_c}. \end{cases} \tag{2.19}$$

Among them, $P_{initial}$ is the initial surviving probability (y-intercept) and $A_{thriving}$ is the moment in the growing process with the best condition; k_s is a coefficient that considers intraspecies competition. When there are many dandelions nearby, the survival rate of one certain dandelion should decrease. R_s is the radius that is set by users, depending on the severity of intraspecies competition. c_c is a coefficient around 1: the smaller c_c is, the more severe the competition is. We call it the *Competition Coefficient*.

Also, since a seed cannot grow if it is eclipsed by an existing dandelion, and the radius of a dandelion plant is 0.3m, we consider all seeds that drop in this range as dead.

We also take precipitation into account because precipitation can promote the growth of dandelions. F_r is a measure of the frequency of rainy days as discussed above. We assume this equation:

$$P_{initial} = P_{initial,max} \cdot (1 - \frac{1}{F_r \cdot 100}) \tag{2.20}$$

Therefore, a survival rate against age function is presented.

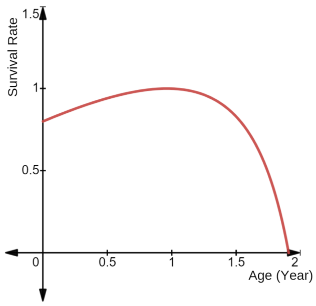


Figure 8: Survival Rate against Age function

2.6 Parameter Calibration and Model Results

2.6.1 Parameter Calibration

The following flow chart summarizes our model used to determine dandelion spread.

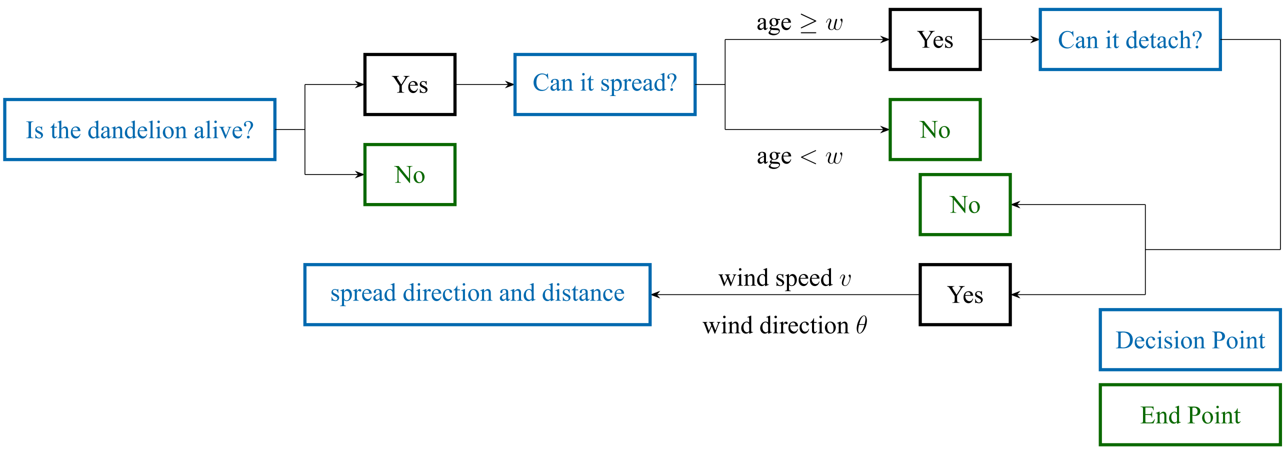


Figure 9: Flow chart for the determination of dandelion spread.

First, regarding precipitation, we set the rainy day frequency F_r in consideration of the climate report in specific locations. For example, for Ames, we calculate the rainy day frequency according to [12]. Furthermore, the ratio between the possibility of detachment in dry and wet conditions C is determined by [5].

While adjusting the wait time, we need to determine a maximum and minimum maturity growth rate. By referring to [13], we assume the following values for v_{min} and v_{max},

$$v_{min} = 100/90 \approx 1.11, \tag{2.21}$$

$$v_{max} = 100/60 \approx 1.67. \tag{2.22}$$

For the environmental factor r_e, we set $m_0 = 0.30$ and $m_1 = 31$ so that a threshold value for rain is reasonably-accounted.

For the number of offspring, we assume each dandelion produces 1000 seeds.

We also assume that dandelion start spreading from the top left corner of the land, the radius R_d of one dandelion is 0.3m, and the radius R_s is 0.6m.

Lastly, while calculating the survival rate function, the initial values $P_{initial}$ and $A_{thriving}$ are chosen as 0.85 and 1.5 (years), respectively. The competition coefficient c_c is chosen as 0.85.

The following figures show our model's prediction of the dandelion's growth after 1, 2, 3, 6, and 10 months. The density and age distribution are given as colored grids, while the annual change in population over time is presented as well.

2.6.2 Model Result

We display the following graphs to show the dispersal of dandelions and overall population throughout 1, 2, 3, 6, and 12 months. The upper graphs display the dispersal, with the x-axis and the y-axis representing the position and color depth representing the population density. The lower graphs display the population, with time on the x-axis and population on the y-axis.

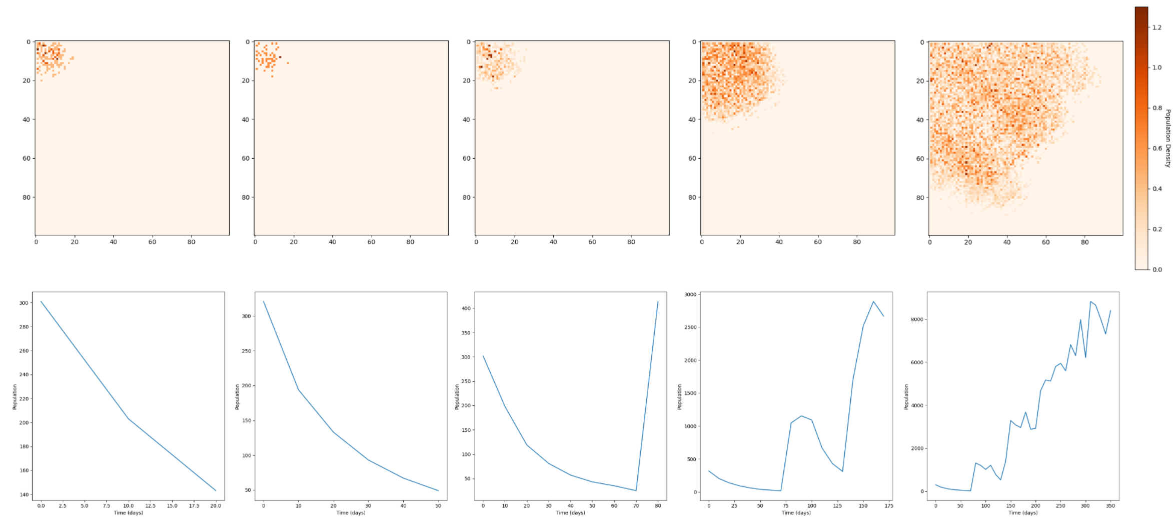


Figure 10: Dispersal and population diagrams throughout 1, 2, 3, 6, and 12 months

Additionally, to prove the validity of our model, we extend the time range to 48 months, which is shown in the following diagrams. After 48 months, the dandelions have completely dominated the land, and their total number fluctuates around a saturated value, which aligns with real-world conditions.

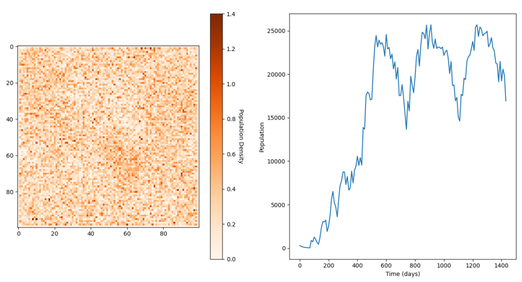


Figure 11: Dispersal and population diagrams over 48 months

2.7 Sensitivity Analysis

2.7.1 Baseline

By default, the annual wind rose data is based on that of Ames (AMW), Iowa, USA. We have chosen data from this station because Ames' wind patterns are typical for dandelion dispersal. Ames has moderate annual temperature and precipitation distributions, providing possibilities for dandelion growth. Moreover, if not specifically pointed out, we assume that the very first dandelion reaches a puffball stage in January, [6]. The precipitation rate is chosen by default as F_{r_0} in Ames. The initial dandelion in its puffball stage is assumed to be placed within the middle of the open plot for better conservation. The initial survival rate and competition coefficient are set at 0.85

Location	Wait Time	Implantation
Ames, Iowa	$v_{max} = 100/60, v_{min} = 100/90$	January
Precipitation	Initial Survival Rate	Competition Coefficient
$F_{r_{original}}$	0.85	0.85

Table 2: Initial values of the model.

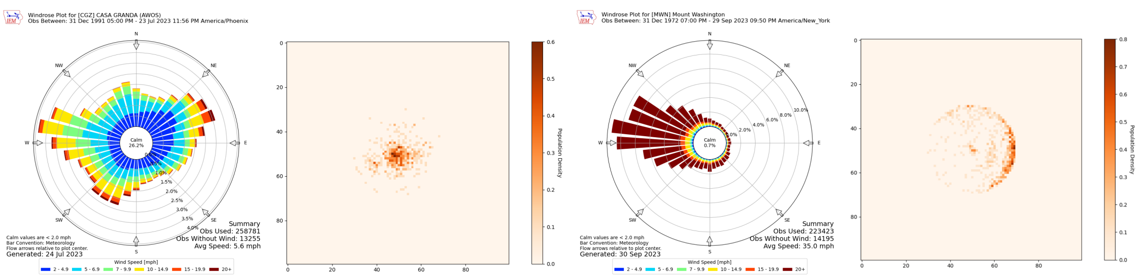
2.7.2 Wind Roses

We take Ames, Iowa as the baseline geographic condition. For sensitivity analysis, we take *Casa Granda* and *Mount Washington* with relatively extreme wind conditions. Plus, the diagrams shown below depict the spread of dandelions after one single dispersal event because the impact of wind roses can be illustrated more thoroughly through short term analysis.

Casa Granda is one of the least windy cities in America, with an average annual wind speed of 5.6 miles per hour as shown in fig. 12 (a). [14]

Mount Washington is the windiest place in America, with an average annual wind speed of 35.0 miles per hour as shown in fig. 12 (b). Dandelions are common invasive species on Mount Washington, [15].

We observe that the model well-aligns with real-world conditions. Seeds can travel farther distances in places with stronger wind. Also, the seeds' main distribution direction aligns with the dominant wind direction, as expected. For example, the seeds in Mount Washington mainly travel eastward for long distances because of the strong wind blowing from the west.

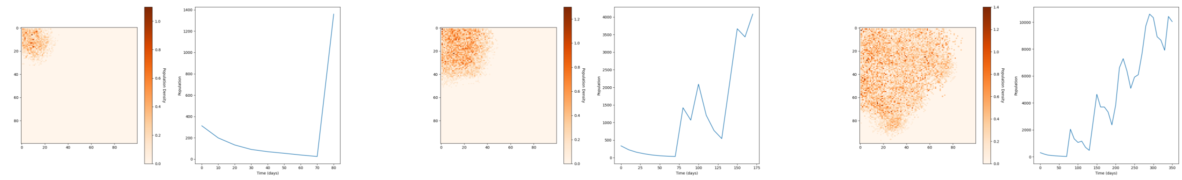


(a) Casa Granda, Arizona (b) Mount Washington, New Hampshire

Figure 12: Wind rose and spread of dandelion in Casa Granda and Mount Washington, as shown in (a) and (b), respectively.

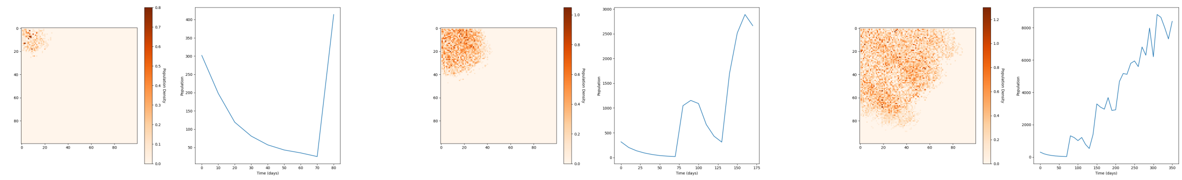
2.7.3 Wait time

We alter v_{min} and v_{max} to determine the effect of different wait time factors on dandelion dispersal. A more apparent change in wait time affected by the implantation date is discussed in the next section. When v_{min} and v_{max} are altered to smaller values, the spreading area decreases dramatically. This can be explained by an increase in wait time resulting in a decrease in dispersal speed. As a result of this decreased speed, the effect of intraspecific competition and natural death becomes apparent, causing a decrease in the spreading area. Results are shown in figs. 13 to 16.



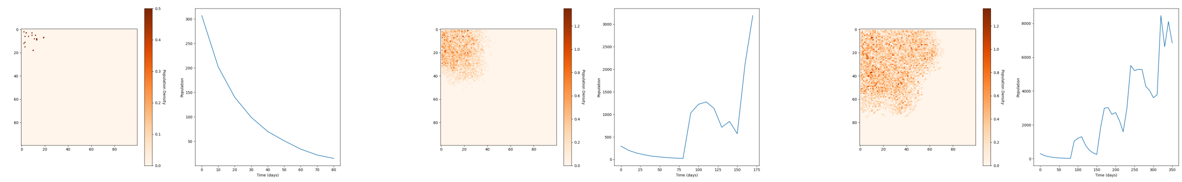
(a) 3 months after implantation (b) 6 months after implantation (c) 12 months after implantation

Figure 13: $v_{min} = 100/80, v_{max} = 100/50$



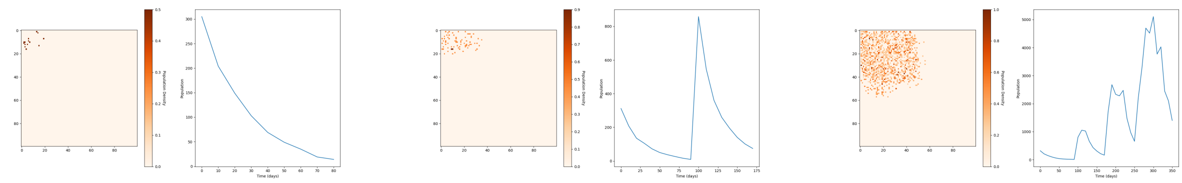
(a) 3 months after implantation (b) 6 months after implantation (c) 12 months after implantation

Figure 14: $v_{min} = 100/90, v_{max} = 100/50$



(a) 3 months after implantation (b) 6 months after implantation (c) 12 months after implantation

Figure 15: $v_{min} = 100/90, v_{max} = 100/60$



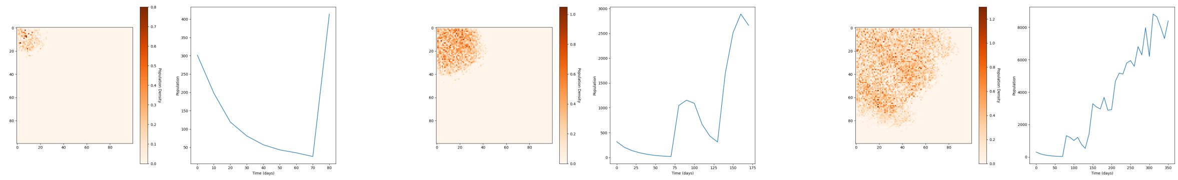
(a) 3 months after implantation (b) 6 months after implantation (c) 12 months after implantation

Figure 16: $v_{min} = 100/100, v_{max} = 100/70$

2.7.4 Implantation Date

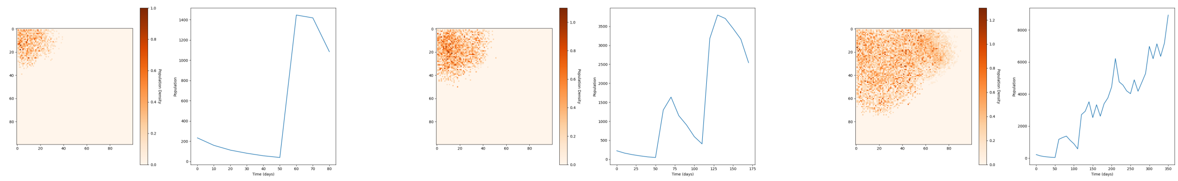
We alter the date at which our first dandelion reaches a puffball stage: the initialization time is chosen in January, April, July, and October, shown in figs. 17 to 20.

Seeds dispersed in January reach a population of 8000 over a year; seeds dispersed in April spread moderately over a year, reaching a population slightly over 8000; seeds dispersed in July display spreading patterns similar to those planted in April; seeds dispersed in October bloom at $12,000$ dandelions over a year. The increase in dispersal area and population density in October implanting can be attributed to the fact that the dispersed dandelions mostly reach a puffball stage around June or July in the next year: October happens to be the golden age after this new generation's dispersal.



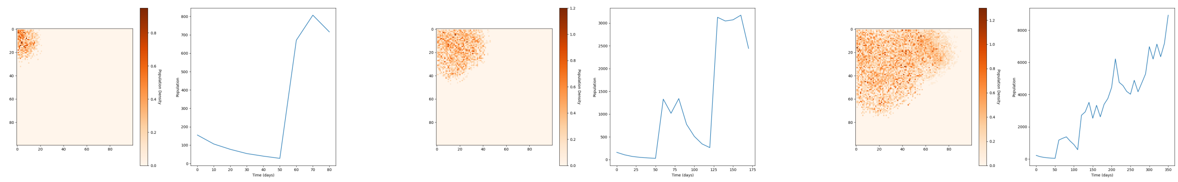
(a) 3 months after implantation (b) 6 months after implantation (c) 12 months after implantation

Figure 17: Implanting in January



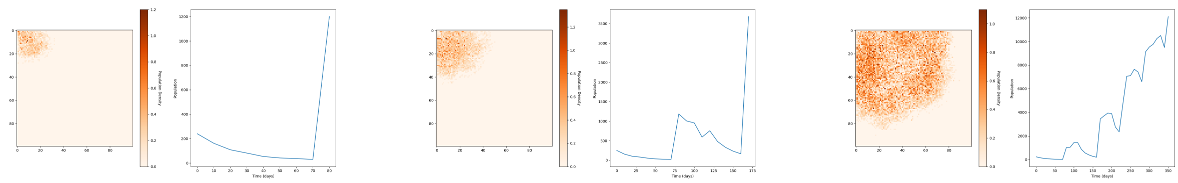
(a) 3 months after implantation (b) 6 months after implantation (c) 12 months after implantation

Figure 18: Implanting in April



(a) 3 months after implantation (b) 6 months after implantation (c) 12 months after implantation

Figure 19: Implanting in July

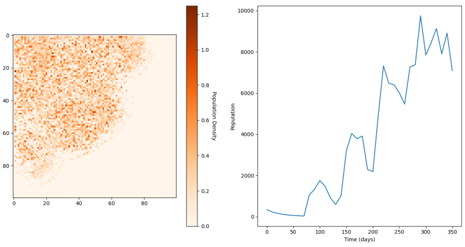


(a) 3 months after implantation (b) 6 months after implantation (c) 12 months after implantation

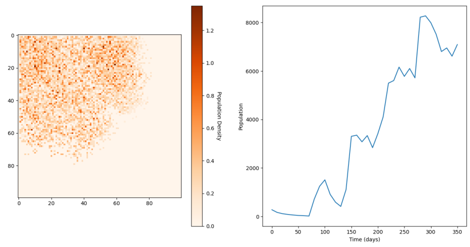
Figure 20: Implanting in October

2.7.5 Precipitation

We assume that the frequency of rainy days in a full month F_r has a fluctuation of $\pm 20\%$. As displayed in fig. 21, compared to dandelions dispersed under normal circumstances, dandelions dispersed in comparatively arid climates seem to reach a higher population over a year, even though their population fluctuates more frequently due to the lack of rain. Such a phenomenon can be attributed to a decrease in pappus detachment caused by less rain. Reversely, when a dandelion is placed within a tropical climate, the dispersal area as well as fluctuations in population decrease. Increased rainfall accelerates the growth in maturity rate, but decreases the number of pappus successfully dispersed.



(a) $F_r = F_{r_0} - 20\%$



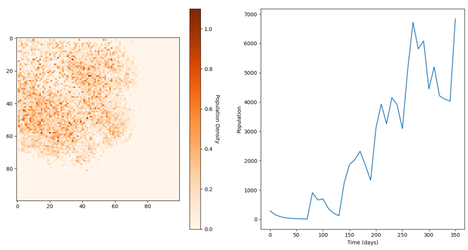
(b) $F_r = F_{r_0} + 20\%$

Figure 21: Model results with different precipitation frequency F_r.

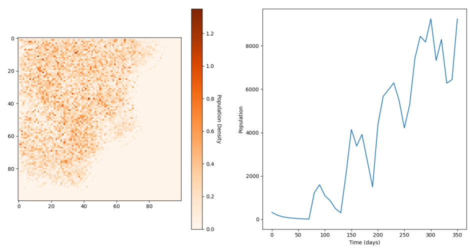
2.7.6 Initial Survival Rate

If the competition coefficient c_c is controlled, the survival rate varies with the initial survival rate. Changing the initial survival rate not only alters the y-intercept of the survival rate against the age diagram but also changes the survival rate for young dandelions. In other words, the lower the P_i, the lower the survival rate for young dandelions is.

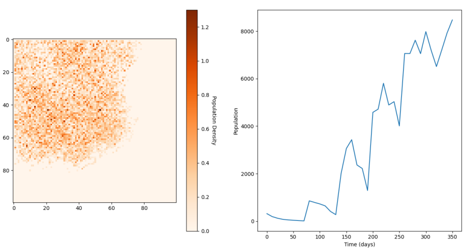
Looking at the following four graphs in fig. 22 where P_i is 0.7, 0.75, 0.8, and 0.9, the first difference is the maximum population dandelions reach. The higher P_i is, the higher the maximum population dandelions can reach. The maximum population happens between 7000, 9000, 8500, and 11500, respectively. The phenomenon is easy to interpret as more dandelions survive.



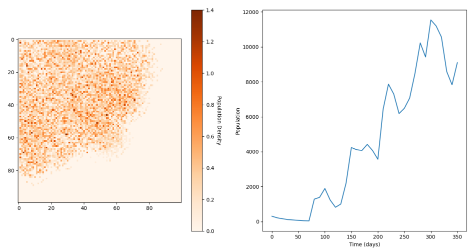
(a) $P_i = 0.7$



(b) $P_i = 0.75$



(c) $P_i = 0.8$



(d) $P_i = 0.9$

Figure 22: Model results with different P_i

However, the survival rate does not affect the distribution and population to a significant extent. This is because we have another factor, competition, weakening the overall survival rate. Since higher P_i can lead to a denser population, many dandelions will die of intraspecies competition, so the overall population will not surge.

Therefore, we conclude that P_i is relatively sensitive, but its effect is weakened by other factors.

2.7.7 Competition Coefficient

We calibrate the dying radius used to determine the competition coefficient c_c, and discover that c_c possesses a dominant control over the dandelion spread. The spreading area and population density vary significantly as the competition factor is changed slightly within 0.80, 0.85, and 0.87, as shown in fig. 23.

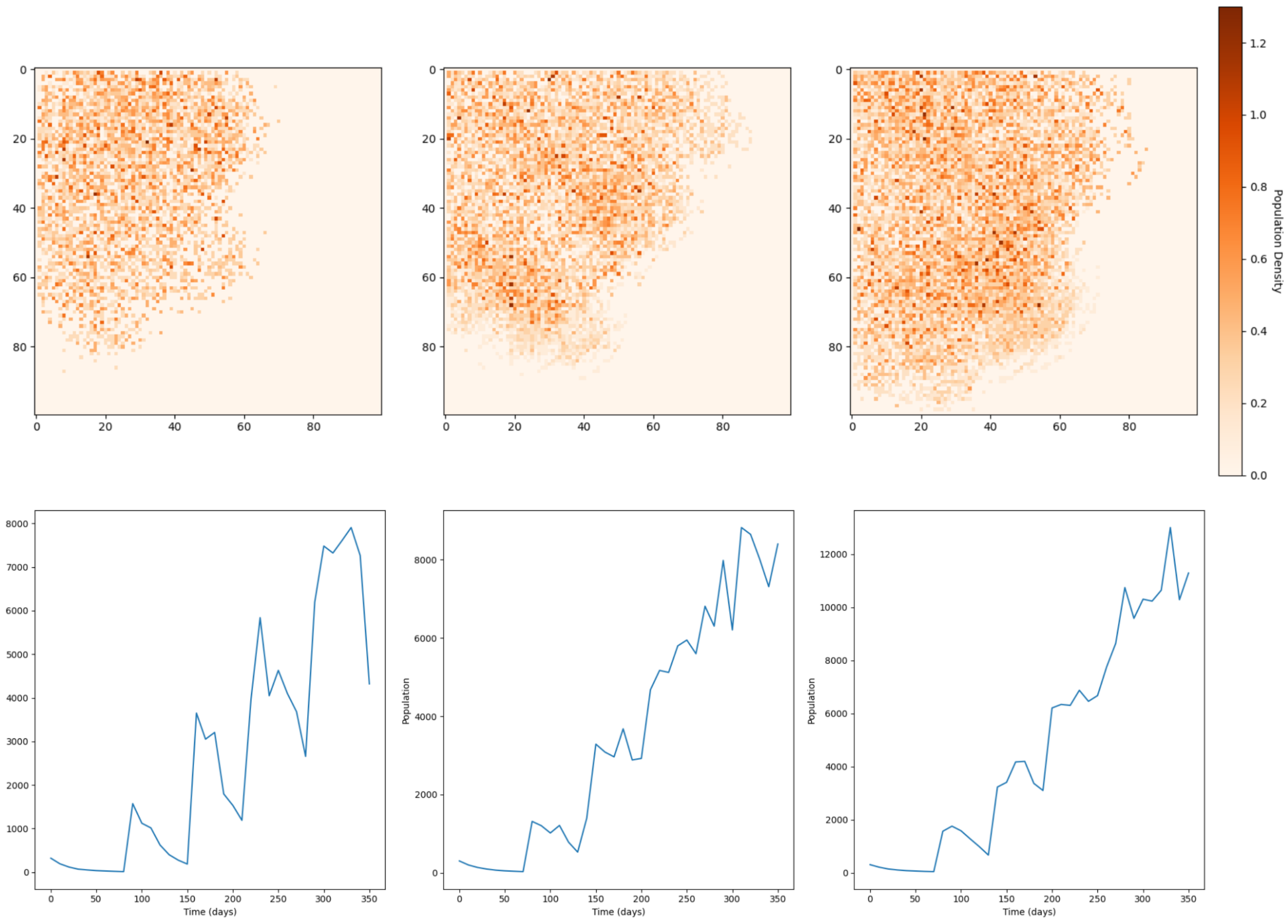


Figure 23: Population density and total population diagram after 12 months of implantation with competition coefficient equal to 0.80, 0.85, and 0.87, respectively.

3 Impact Factor for Invasive Species

Taraxacum officinale, originally native to Europe and Asia, was imported to America as a food. However, due to its strong ability to spread and reproduce, it spreads quickly to different parts of the world and is sometimes considered as an invasive species in specific regions. To assess the invasiveness of dandelions, we introduce an evaluation model.

3.1 Method

In our model, TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) is applied. TOPSIS is a multi-criteria decision model that is relatively simple and flexible. In TOPSIS, the inputs are the data under each criterion for different evaluation objects, and the outputs are the scores of each evaluation object.

The model mainly includes these following steps:

- **STEP 1**: Create a matrix $(X_{ij})_{mn}$composed of m evaluation objects and n criteria, where i and j represent the row number and column number, respectively.

$$\begin{pmatrix} X_{1,1} & X_{1,2} & \dots & X_{1,n} \\ X_{2,1} & X_{2,2} & \dots & X_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{m,1} & X_{m,2} & \dots & X_{m,n} \end{pmatrix} \tag{3.1}$$

- **STEP 2**: Normalize the matrix to $(Z_{ij})_{mn}$ with the formula $Z_{ij} = \dfrac{X_{ij}}{\sqrt{\sum_{k=1}^{n}(X_{ij})^2}}$
- **STEP 3**: Determine the best and worst value in each criteria $Z_j^+ = \max\{z_{i,j} \mid i = 1, 2, \dots, n\}$, $Z_j^- = \min\{z_{i,j} \mid i = 1, 2, \dots, n\}$.
- **STEP 4**: Calculate the distance between the target value and the best and worst value with formula $D_i^+ = \sqrt{\sum_{j=1}^{m} W_j \cdot (Z_j^+ - z_{ij})^2}$ and $D_i^- = \sqrt{\sum_{j=1}^{m} W_j \cdot (Z_j^- - z_{ij})^2}$, where W_j is the weight of each criteria.
- **STEP 5**: Evaluate the object's closeness to the best value with formula $C_i = \dfrac{D_i^-}{D_i^+ + D_i^-}$

Therefore, the C_i value will be the "impact factor" we aim to obtain. There are some other details that will be discussed in the following paragraphs.

3.2 Criteria and Data Selection

In order to evaluate an "impact factor" for invasive species, we must compare its invasiveness with other plants. Therefore, we need to first create a data set for various plant species. To obtain a fair number, the plants we select should vary from non-invasive to extremely invasive. The plants we select are *rose, sunflower, marigold, strawberry bush, water milfoil, bamboo, and kudzu*.

Based on the research we analyzed [16, 17], a few criteria are selected to evaluate a plant's invasiveness:

- **Relative Growth Rate (RGR)** is a measure of growth rate relative to size. It can be obtained by the formula $RGR = \frac{\ln S_2 - \ln S_1}{t_2 - t_1}$ with the unit gg^{-1}day^{-1}. A plant's RGR changes overtime, but in this model, we take its average value. This variable is chosen as a criterion because most invasive species grow very rapidly. The higher this value is, the more invasive a plant is considered to be.

- **Wait Time** refers to the duration between a plant's initial growth or germination date and the time it reaches maturity. The wait time can vary, so we take their average values. This variable represents how fast a species can spread and reproduce. The smaller this value is, the more invasive a plant is considered to be.

- **Successful Offspring Number (SON)** means the number of seeds a plant can produced that can successfully germinate. This variable also represents how quickly a certain species can spread and how strong they in living ability. The higher this value is, the more invasive a plant is considered to be.

- **Adaptability** is a value calibrated by the users on a scale of 0-10, measuring how well a species can adapt to a new environment. When deciding the value, users should first do research about a certain plant and then evaluate the number. The higher this value is, the more invasive a plant is considered to be.

- **Damage to Environment** is a value calibrated by the users on a scale of 0-10, measuring the extent of damage a plant can cause to an ecological system. When evaluating the value, users must conduct research on the plant's various characteristics, such as its nutrient consumption, impact on local animal and plant populations, toxicity, resistance to elimination, and so on.

We estimate the values of these criteria by conducting research in various ways. Many of our data are obtained from [18]. We also add the data of *dandelion, Japanese knotweed, and Himalayan Balsam* to test the viability of this model. *Japanese knotweed and Himalayan Balsam* are the two invasive species we chose, and they will be analyzed later in details.

Species	RGR	w (day)	SON	Adaptability	Damage Rate
Rose	0.02	180	10	4	2
Sunflower	0.1	100	500	6	2
Marigold	0.05	120	20	7	2
Strawberry Bush	0.04	150	200	5	3
Water milfoil	0.05	365	4	7	8
Bamboo	0.6	365	1	8	5
Kudzu	0.8	90	30	10	10
Dandelion	0.3	90	800	9	5
Japanese knotweed	1	60	500	9	8
Himalayan Balsam	0.6	90	400	8	7

Table 3: Data set containing raw values of different criteria for various invasive species.

3.3 Model Results

Before running the TOPSIS algorithm, there are some adjustments we have to make.

To begin, we must align the trend of each criterion. Since w is the only negative index, we convert it into a positive index by $w' = w_{max} - w$.

Then, we have to determine W_j, namely, the weight for each criterion. In our model, we consider the weights to be the same because the entropy weighting method, the traditional way to calculate weights, does not work well in our case. Weights obtained by the entropy weighting method are calculated through the variation of data in each criterion, which can be very different in our data set. Therefore, we consider all W_j to be equal because each criterion is equally important.

Running the model, we obtain this result in tab. 4.

The C_i is the "impact factor" we obtain. For non-invasive species, the number is around 0.3, while for very invasive species, the number is around 0.7. In the case of dandelions, C_i is 0.63, indicating that dandelions are relatively invasive. However, when compared with highly invasive species, the C_i for dandelions suggests that dandelions are not as invasive. This aligns with real-world conditions, where dandelions are considered widespread but not totally invasive.

The two species we choose to test our model are *Japanese knotweed* and *Himalayan Balsam*.

Japanese knotweed is native to Asia and was introduced to the United Kingdom in 1825 [19]. They usually invade places with high light, such as roadsides. They are considered highly invasive

Species	D_i^+	D_i^-	C_i	Rank
Rose	2.03	0.61	0.23	10
Sunflower	1.56	1.12	0.41	5
Marigold	1.78	0.95	0.35	7
Strawberry Bush	1.75	0.78	0.31	9
Water milfoil	1.80	0.90	0.33	8
Bamboo	1.63	0.97	0.37	6
Kudzu	0.99	1.86	0.65	2
Dandelion	0.97	1.65	0.63	4
Japanese knotweed	0.48	1.91	0.80	1
Himalayan Balsam	0.82	1.50	0.65	3

Table 4: Model Result: Impact factor C_i for different plants

because they spread seeds rapidly and are extremely hard to eradicate. The result of our model proves that they are very invasive, with an extremely high C_i of 0.80.

Himalayan Balsam is native to Himalayan and is now one of the UK's most invasive weeds, [20]. They thrive near riverbanks and can grow to 1-3m, blocking the light to native plants, [21]. They also have large offspring numbers and short offspring periods, contributing to their invasiveness. The result of our model proves that they are invasive, with a high C_i of 0.65.

4 Conclusion

In conclusion, we have developed a mathematical model to predict the spread of dandelions over time. The model incorporates various environmental factors to accurately simulate real-world conditions. Additionally, we have designed an evaluation model to quantify dandelions and other plants' invasiveness. The models have some strengths that enhance their accuracy and some limitations that weaken their preciseness.

4.1 Strength

Our model successfully incorporates the effect of various climatic factors on the growth of dandelions.

- The utilization of wind roses: Our employment of wind roses allows us to account for the different effects of wind on dandelion dispersal. We can identify data detailed to the distribution of monthly wind direction as well as monthly wind speed. The obtainment of these data allows us to determine the spread of dandelions with better accuracy and precision.

- The consideration of various factors: We considered the effect of precipitation, temperature, and timing. Regarding precipitation, we not only took into account of the effect of rain on puffball detachment, but also considered the positive effect of rain on maturity rate growth. This allows for a detailed model that allows realistic interpretations.

- The employment of a normal distribution function: Our use of a normal distribution function to calibrate dandelion wait time helps us to incorporate the effect of temperature and other climatic factors on dandelion growth. The control variables v_{min} and v_{max} can be effectively utilized to control the growth in maturity rate.

- The calibration of a survival rate function: In our model, we considered the survival rate of individual dandelions as a function to time. Indeed, the survival rate is not permanently-determined once a seed has reached the ground; it evolves as surrounding factors change. Such predictions are much more accurate than predetermined or randomly-assigned survival rate models.
- The consideration of intraspecific competition: The competition between nearby dandelions are incorporated in the maturity rate and survival rate of our model. In the determination for the maturity rate, we took into account of the number of surrounding plants and their distance to the dandelion of our interest. In the determination for the survival rate, a different radius is specified: any seeds dispersed within the circle centered at some dandelion with that radius immediately dies.
- The use of the TOPSIS model: unlike regular evaluation models which possess a great dependence on the tester's subjectivity, the TOPSIS model incorporates subjective and objective evaluation parameters.

4.2 Limitations

- Our constructed model involves numerous variables, adding to the intricacy of computations. It will be better if we can design a more elegantly-refined solution.
- We did not delve into the direct connection between the coefficient and the intensity of intraspecies competition about nearby plants, as there is a dearth of field data. The value is determined using an approximation.
- When assessing the impact factor of invasive species, certain data lacks precision and is averaged. Additionally, the determination of values for some parameters is less objective.

References

[1] Engineers uncover secret ‘thinking’ behind dandelions’ seed dispersal | Imperial News | Imperial College London — imperial.ac.uk. https://www.imperial.ac.uk/news/236934/engineers-uncover-secret-thinking-behind-dandelions/.

[2] Ikuo Kandori, Toshihiro Hirao, Satoshi Matsunaga, and Tsutomu Kurosaki. An invasive dandelion unilaterally reduces the reproduction of a native congener through competition for pollination. *Oecologia*, 159:559–569, 2009.

[3] IEM: Site Wind Roses — mesonet.agron.iastate.edu. https://mesonet.agron.iastate.edu/sites/windrose.phtml?station=AXA&network=IA_ASOS.

[4] Zoe Bont, Marc Pfander, Christelle AM Robert, Meret Huber, Erik H Poelman, Ciska E Raaijmakers, and Matthias Erb. Adapted dandelions trade dispersal for germination upon root herbivore attack. *Proceedings of the Royal Society B*, 287(1921):20192930, 2020.

[5] Madeleine Seale, Oleksandr Zhdanov, Cathal Cummins, Erika Kroll, Michael Blatt, Hossein Zare-Behtash, Angela Busse, Enrico Mastropaolo, Ignazio Maria Viola, and Naomi Nakayama. Moisture-dependent morphing tunes the dispersal of dandelion diaspores. *Available at SSRN 3334428*, 2019.

[6] Alex Gee. How plants are adapting to warmer growing seasons - Global Center on Adaptation — gca.org. https://gca.org/how-plants-are-adapting-to-warmer-growing-seasons/.

[7] Mohammed H Abu-Dieyeh, Jerome Bernier, and Alan K Watson. Sclerotinia minor avances fruiting and reduces germination in dandelion (taraxacum officinale). *Biocontrol Science and Technology*, 15(8):815–825, 2005.

[8] O Kenny, NP Brunton, D Walsh, CM Hewage, P McLoughlin, and TJ Smyth. Characterisation of antimicrobial extracts from dandelion root (taraxacum officinale) using lc-spe-nmr. *Phytotherapy Research*, 29(4):526–532, 2015.

[9] Brian J Iaffaldano, Yingxiao Zhang, John Cardina, and Katrina Cornish. Genome size variation among common dandelion accessions informs their mode of reproduction and suggests the absence of sexual diploids in north america. *Plant Systematics and Evolution*, 303:719–725, 2017.

[10] E Estrelles, E Biondi, M Galiè, F Mainardi, A Hurtado, and P Soriano. Aridity level, rainfall pattern and soil features as key factors in germination strategies in salt-affected plant communities. *Journal of Arid Environments*, 117:1–9, 2015.

[11] China’s seventh census data. http://www.stats.gov.cn/sj/pcsj/rkpc/7rp/indexch.htm.

[12] Average monthly rainy days in Chicago (Illinois), the United States of America — weather-and-climate.com. https://weather-and-climate.com/average-monthly-Rainy-days,Chicago,United-States-of-America.

[13] Elmer Gray, Eugene M McGehee, and Don F Carlisle. Seasonal variation in flowering of common dandelion. *Weed Science*, 21(3):230–232, 1973.

[14] Top 10 Least Windy Cities in the US, My Top Global — mytopglobal.com. https://mytopglobal.com/least-windy-cities/.

[15] David Brooks. An invasive species on Mount Washington: Dandelions. (Dandelions??) - Granite Geek — granitegeek.concordmonitor.com. https://granitegeek.concordmonitor.com/2017/04/27/invasive-species-mount-washington-dandelions-dandelions/.

[16] Karin M Kettenring and Carrie Reinhardt Adams. Lessons learned from invasive plant control experiments: a systematic review and meta-analysis. *Journal of applied ecology*, 48(4):970–979, 2011.

[17] Marcel Rejmánek. Invasive plants: approaches and predictions. *Austral ecology*, 25(5):497–506, 2000.

[18] ResearchGate | Find and share research — researchgate.net. `https://www.researchgate.net/`.

[19] Japanese knotweed, Reynoutria japonica Polygonales: Polygonaceae - EDDMapS — eddmaps.org. `https://www.eddmaps.org/species/subject.cfm?sub=19655`.

[20] Himalayan balsam - Invasive Species — invasive-species.org. `https://www.invasive-species.org/species/himalayan-balsam/`.

[21] Himalayan Balsam – Profile and Resources | Invasive Species Centre — invasivespeciescentre.ca. `https://www.invasivespeciescentre.ca/invasive-species/meet-the-species/invasive-plants/himalayan-balsam/`.