INTEGIRLS Shanghai Spring 2025

Relay Round 2 10 minutes

Question 1 (5 points)

Given $a > 0$, $b > 0$ and $a + b = 2$, find the maximum of $\dfrac{a^2 + b}{a^2 + b^2}$. Your answer should be in the form of $\dfrac{a + \sqrt{b}}{c}$, where a, b, c are integers and b is not divisible by the square of any prime. Pass on the integer $a + b + c$.

给定 $a > 0$，$b > 0$ 且 $a + b = 2$，求 $\dfrac{a^2 + b}{a^2 + b^2}$ 的最大值。你的答案应为 $\dfrac{a + \sqrt{b}}{c}$ 的形式，其中 a, b, c 为整数，且 b 不能被任何素数的平方整除。给出整数 $a + b + c$。

Question 2 (7 points)

Suppose $T = TNYWR$. Let p, q be positive integers such that $1 \le p < q \le T$, $\gcd(p, q) = 1$, and $p + q > n$. If $\sum \dfrac{1}{pq} = \dfrac{m}{n}$ in simplest form, what is $m + n$?

设 $T = TNYWR$。令 p, q 为正整数，满足 $1 \le p < q \le T$，$\gcd(p, q) = 1$，且 $p + q > n$。如果 $\sum \dfrac{1}{pq} = \dfrac{m}{n}$ 为最简形式，求 $m + n$ 的值。

Question 3 (7 points)

Let $T = TNYWR$. You are given $2T$ plus one steps and $2T$ minus one steps to arrive at a 0 in the end without going negative during the sequence of calculations. How many valid sequences are there where the cumulative sum equals 0 at least once during the sequence?

$T = TNYWR$. 你有 $2T$ 次 “+1” 操作和 $2T$ 次 “-1” 操作，你需要在计算过程中不出现负数的情况下，使得最终得到的结果为 0。有多少种有效的序列，使得累积和在计算过程中至少有一次等于 0?

Question 4 (12 points)

Let $T = TNYWR$. How many positive integers of the form $k = 2^a 3^b$ are there such that a, b are non-negative integers and $k \le T$?

设 $T = TNYWR$。有多少个正整数 $k \le T$ 满足 $k = 2^a 3^b$，其中 a, b 为非负整数?

END OF TEST.